Factor exposures of smart beta indexes
Introduction

Capitalisation weighted indexes are considered to be representative of the broad market opportunity set and are characterised by high levels of liquidity, investment capacity and relatively low levels of turnover. However, concentration risks that may arise during market bubbles and the inherent factor traits of capitalisation weighted indexes have prompted alternative approaches to index construction, with the resulting indexes commonly referred to as “smart beta”. Smart beta indexes encompass both alternatively weighted and factor indexes.

Alternatively weighted indexes have a variety of objectives. For example, equally weighted indexes achieve increased diversification by allocating capital equally across constituents. Fundamental indexes weight stocks by their economic scale, resulting in a weighting scheme that is independent of price-based market measures of size. Risk-based alternatively weighted indexes are typically based on a Markowitz optimisation, yet their objectives may be distinct: minimum variance indexes aim to achieve volatility reduction relative to the capitalisation weighted benchmark; equal risk contribution indexes are designed to improve risk diversification; and risk-efficient indexes aim to maximise the expected Sharpe Ratio.

An incidental outcome of alternative weighting approaches is a sometimes unintentional and/or uncontrolled exposure to risk factors. And while alternatively weighted indexes are intended to achieve certain objectives, they are also frequently used as a means of capturing the long-term risk premia associated with various factors. An alternative way of capturing factor risk premia is via diversified, transparent and replicable factor indexes, which are designed to exhibit controlled exposure to the factor or factors of interest.

In this report, we discuss ways of defining and calculating factor exposure. We demonstrate the factor exposures implicit in a range of alternatively weighted indexes and contrast them with the factor exposures of comparable factor indexes (i.e., indexes with explicit factor objectives). The results suggest that whilst alternatively weighted indexes do exhibit factor exposures, these exposures may be relatively weak, time-varying and are often diluted by exposure to non-target factors. We demonstrate that factor indexes, in a general framework, are better suited for capturing desired exposures than alternatively weighted indexes, particularly in the context of multiple factor objectives.

We also highlight the importance of taking a holistic approach when seeking multiple factor objectives. Simply aggregating single factor indexes may defeat the objective of achieving substantial exposure to multiple factors, since a combination of negatively correlated single factors can result in an undesired dilution of the intended factor exposures. For example, a simple combination of a value index and a quality index may result in relatively low exposure to both factors. We propose an alternative approach, which results in greater exposure to both factors. We demonstrate that an index with exposure to a particular factor (for example, value) can achieve additional factor objectives by tilting from the existing position towards those objectives (for example, towards quality). The resulting index preserves its original value focus while also offering exposure to quality.
1. Alternatively weighted indexes and factor indexes

1.1 Alternatively weighted indexes

Alternatively weighted indexes were originally developed to address the concentration risks that may arise periodically in capitalisation weighted indexes. For example, during periods such as the technology, internet and telecommunications bubble of 1999-2000, capitalisation weighted indexes became highly concentrated in particular stocks and sectors.

The most intuitive alternative to a capitalisation weighted index is an equally weighted index. By comparison with capitalisation weighting, an equally weighted index achieves enhanced diversification by allocating equal amounts of capital to each index constituent. From a factor perspective, an equally weighted index displays a bias towards the size factor, since by comparison with a capitalisation weighted index an equally weighted index allocates larger weightings to small capitalisation stocks (the factor exposures of alternatively weighted indexes are discussed at length in Section 3 of this paper).

Minimum variance indexes are constructed with the objective of minimising index risk by means of a mathematical optimisation, using stocks’ historical volatility and correlations as inputs. Unconstrained formulations of the minimum variance approach (e.g., Black et al., 1972) display high levels of concentration. Most index providers employ constraints in order to limit the degree of concentration in individual countries, sectors and stocks. In the FTSE Global Minimum Variance Index Series, FTSE incorporates an explicit diversification constraint to mitigate concentration concerns.

In an equal risk contribution (ERC) index, the diversification objective is achieved by recognising that equal weighting does not result in equal contributions to index-level risk (since stocks have different volatilities and correlations). Instead, the ex-ante contribution to index-level risk of each index constituent is equalised by means of an optimisation. Empirical studies (FTSE, 2013) illustrate that the ex-post risk contribution of each stock is also approximately equal. The concentration and volatility levels of an ERC index lie between those of an equally weighted and a minimum variance index. From a factor perspective, ERC indexes therefore exhibit both size and low volatility factor exposures.

Risk-efficient indexes (such as the FTSE EDHEC-Risk Efficient Index Series) aim to improve the capitalisation weighted index’s risk/return trade-off by positing a positive relationship between downside risk and expected return. The resulting optimisation results in an index with the highest expected return per unit of volatility (i.e., the maximum expected Sharpe Ratio).

Fundamental indexes use a company’s economic footprint (as opposed to market capitalisation) to determine constituent weightings. The FTSE RAFI Index Series use sales, cash flow, book value and dividends as measures of company size. Fundamental indexes share many desirable characteristics of capitalisation weighted indexes, such as relatively high levels of liquidity and investment capacity. The non-price-based weighting scheme suggests that they are less susceptible to bubbles or momentum-driven market excesses.

Equally weighted, risk based and fundamental indexes all attempt to overcome perceived drawbacks as compared to capitalisation weighted indexes. Table 1 summarises these alternatively weighted indexes and their objectives.
An incidental outcome of alternative weighting approaches is exposure to common risk factors that have historically been associated with positive long run risk premia. While alternatively weighted indexes achieve their initial objectives, they are also frequently used as a means of capturing risk premia. However, the increasing interest in factors has resulted in the creation of diversified, transparent and replicable factor indexes, which are designed to exhibit controlled and explicit exposure to single or multiple factors.

### 1.2 Factor Indexes

Factor indexes have the specific objective of providing exposure to a particular factor or factors. For example, the FTSE Global Factor Index Series aims to achieve controlled exposure to specific stock-level characteristics, such as size, value and momentum (see FTSE, 2014a), with due consideration for the diversification and capacity characteristics of the resulting factor index.

To highlight the distinction between alternatively weighted and factor indexes, consider the characteristics of a minimum variance index (a popular alternative weighting methodology) and a low volatility factor index. While the minimum variance index has the objective of minimising volatility, the low volatility factor index aims to capture the factor premium associated with low volatility stocks, with relatively small incidental reductions in index-level volatility. The distinction between a FTSE Minimum Variance index and a FTSE Low Volatility Factor Index is discussed at greater length in Section 3.

The weights of a FTSE Factor Index are determined in the following manner. For stocks in an underlying index universe \( U \), with underlying index weights \( W_i \), the factor index weights are:

\[
\hat{W}_i = \frac{S_i \cdot W_i}{\sum_{j \in U} S_j \cdot W_j} \quad \text{.................................................. (1)}
\]

Where \( S_i \) is the standard cumulative normal distribution function of the cross-sectional Z-Scores for a given factor.

Formula (1) amounts to the application of a tilt towards the factor of interest, where the strength and direction of the tilt may be varied (see FTSE 2014a for further information on the construction methodology for FTSE’s Global Factor Indexes).

The underlying index weights \( W_i \) may reflect any chosen weighting methodology (e.g., capitalisation, equal, or risk-based). However, a practical choice of underlying index in formula (1) is the capitalisation weighted index. Asness (2006), shows that when the underlying index is capitalisation weighted and \( S \) is the dividend yield, formula (1) results in an index that is similar to a fundamental index, constructed using dividends.
Exposure to multiple factors may be achieved by creating a composite of the factors of interest and the application of formula (1) or by the sequential or repeated application of a series of single factor tilts (i.e., the product of several $S_i$). When applied to the underlying index weights, this sequential tilting results in an index that is tilted first towards one factor and then towards another. The result is that the final index exhibits both factor characteristics. Let $S_{1i}$ and $S_{2i}$ be the standard cumulative normal of cross-sectional Z-Scores of factors 1 and 2 of the $ith$ stock. Then the weight of this stock in the twice tilted index is given by:

$$
\hat{W}_i = \frac{S_{1i} \cdot S_{2i} \cdot W_i}{\sum_{j \in U} S_{1j} \cdot S_{2j} \cdot W_j} \text{................................. (2)}
$$

In Section 2 we discuss approaches to assessing factor exposures. In Sections 3 and 4 we examine the factor exposures of selected alternatively weighted and factor indexes, highlighting the exposure to both target and to non-target (incidental) factors. In Section 5 we examine approaches to achieving simultaneous exposure to multiple factors.

2. Assessing factor exposure

The calculation of an index's factor exposure requires the knowledge of stocks' weightings through time.

Typically, raw factor scores are normalised by removing the sample mean and dividing by the sample standard deviation. The distribution of the resulting standardised factor scores (or "Z-Scores") then has a mean of zero and a standard deviation of one.

The exposure of an index to a given factor may be derived as the aggregate of the individual stock weights, multiplied by the Z-Score of each stock. Since the factor score is standardised at each point in time, meaningful comparisons can be made regarding factor exposure through time. Let $X$ be the exposure of an index to the factor of interest, and $W_j$ and $Z_j$ the index weight and the Z-Score of stock j with respect to the same factor:

$$
X = \sum_{j \in U} W_j \cdot Z_j \text{................................................. (3)}
$$

Active factor exposure is defined as the difference between the index factor exposure and the underlying index factor exposure.

Factor exposure can also be assessed using a returns based analysis. Index excess returns are regressed over some period of time against the returns to a set of standard factor portfolios. The factor exposures, which we refer to as loadings, are then the beta regression coefficients. This is particularly useful approach when holdings based information is unavailable.

However there is an inherent ambiguity in this approach. First, what period of time should be used to make the assessment? Surely it should span a variety of market conditions so as not to bias results. However apart from “the longer the better” no consensus exists. More seriously what is the standard set of factors or factor portfolios? Should we use the Fama and French three factors (Fama and French (1992)), the Carhart four factors (Carhart (1997)) or factors derived from a set of factor mimicking portfolios (Grinold and Kahn (1995))? The choice of factor definitions, portfolio construction technique and factors themselves will all affect the assessment of factor loadings.
Our view is that when holdings information is available exposure should be calculated as in equation (3). It is unambiguous and furthermore can be used to assess factor exposure at a point in time.

### 3. Factor exposure of alternatively weighted indexes

In this section we follow the approach discussed in Section 2 to measure the factor exposures of a set of alternatively weighted indexes. Factor exposures are calculated using the factor definitions set out in the FTSE Global Factor Index Series Ground Rules (see FTSE, 2014e) and Appendix E.

#### 3.1 Equally Weighted Indexes

In Table 2 we examine the average factor exposures of an equally weighted version of the FTSE Developed Index relative to the capitalisation weighted FTSE Developed Index from September 2001 through July 2015.

The size, volatility and illiquidity exposures of the equally weighted index are unsurprising. An equally weighted index is underweight large-cap stocks, leading to a positive exposure to size. Market capitalisation and the size factor are highly correlated with liquidity; consequently, a tilt towards small capitalisation stocks also manifests itself as exposure to relatively illiquid stocks. Furthermore, smaller stocks tend to have higher observed levels of volatility (i.e., a negative exposure to low volatility).

![Table 2. Average active factor exposure – Equally Weighted Index](image)

<table>
<thead>
<tr>
<th></th>
<th>Average active exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Illiquidity</td>
</tr>
<tr>
<td>FTSE Developed</td>
<td></td>
</tr>
<tr>
<td>Equally Weighted</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Table 2. Average active factor exposure – Equally Weighted Index

Source: FTSE Russell. Data as at August 2015. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the final page for important legal disclosures.

FTSE Developed and FTSE Developed Equally Weighted indexes, September 2001–July 2015. The equally weighted index is rebalanced quarterly. Factor exposure is the average monthly exposure as detailed in Section 2.1. A complete set of FTSE Developed Equally Weighted Index factor exposure charts is shown in Appendix A. Past performance is no guarantee of future results. Please see the final page for important legal information.

#### 3.2 Fundamental Indexes

Table 3 exhibits the average factor exposure of the FTSE RAFI Developed 1000 Index\(^1\) relative to the FTSE Developed Index from March 2007 through July 2015. Fundamental company size is correlated with market capitalisation, leading to a negligible exposure to the illiquidity and size factors.

The value (and momentum) exposure of fundamental indexes is unsurprising. Asness (2006) provides one perspective of fundamental indexes as a value tilt on a capitalisation weighted index. Value stocks are frequently out of favour and consequently typically exhibit poor momentum. Quality stocks tend to be expensive; hence, a fundamentally weighted index will also display negative quality exposure.

\(^1\) The FTSE RAFI Developed 1000 Index contains a number of small-cap stocks (around 1-5% of the index by weight) for which factor values were unavailable. We assume the factor Z-scores for these stocks are zero.
Table 3. Average active factor exposure – Fundamental indexes

<table>
<thead>
<tr>
<th>Factor</th>
<th>Illiquidity</th>
<th>12-month momentum</th>
<th>Quality</th>
<th>Size</th>
<th>Value</th>
<th>(Low) volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTSE RAFI Developed 1000</td>
<td>-0.03</td>
<td>-0.11</td>
<td>-0.19</td>
<td>-0.06</td>
<td>0.32</td>
<td>-0.08</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at August 2015. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the final page for important legal disclosures. FTSE Developed and FTSE RAFI Developed 1000 indexes, March 2007–July 2015. The FTSE RAFI Developed 1000 index is rebalanced annually in March. Factor exposure is the average monthly exposure as detailed in Section 2.1. Past performance is no guarantee of future results. Please see the final page for important legal information.

The value and quality exposures of the fundamental index vary through time (see Figures 1 and 2). For example, as equity valuations became increasingly depressed during the 2008 financial crisis, the fundamental index became increasingly value-orientated (Figure 1). During the same period, the increased value exposure was juxtaposed with increasingly negative quality exposure (Figure 2).

Figure 1. Active Value Exposure of FTSE RAFI Developed 1000 Index

Figure 2. Active Quality Exposure Developed 1000 Index

Source: FTSE Russell. Data as at August 2015. FTSE Developed and FTSE RAFI Developed 1000 indexes, March 2007–July 2015. The FTSE RAFI Developed 1000 index is rebalanced annually in March. A complete set of FTSE RAFI Developed 1000 factor exposure charts is provided in Appendix. Past performance is no guarantee of future results. Please see the final page for important legal information.

3.3 Risk-Based Indexes

In Table 4 we show the average monthly active factor exposures of three risk-based indexes relative to the FTSE Developed Index from September 2003 through July 2015. Risk-based indexes (in particular the FTSE Developed Minimum Variance Index and FTSE EDHEC-Risk Efficient Developed Index) exhibit a substantial positive exposure to the size and illiquidity factors.

However, the risk-based indexes display variable exposure to the volatility factor: the FTSE Developed Minimum Variance Index in particular has a positive exposure to the low volatility factor, while the FTSE EDHEC-Risk Efficient Developed index exhibits a negative exposure (i.e., it is exposed to stocks exhibiting above-average levels of volatility). This is possibly the result of the index’s specification of expected return as a positive function of downside risk.
Table 4. Average Active Factor Exposure – Risk-Based Indexes

<table>
<thead>
<tr>
<th>Average active exposure</th>
<th>Illiquidity</th>
<th>12-month momentum</th>
<th>Quality</th>
<th>Size</th>
<th>Value</th>
<th>(Low) volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTSE Developed Equal Risk Contribution Index</td>
<td>0.63</td>
<td>-0.03</td>
<td>0.06</td>
<td>0.71</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>FTSE Developed Minimum Variance Index</td>
<td>1.03</td>
<td>0.00</td>
<td>0.12</td>
<td>1.24</td>
<td>0.04</td>
<td>0.23</td>
</tr>
<tr>
<td>FTSE EDHEC-Risk Efficient Developed Index</td>
<td>0.83</td>
<td>0.05</td>
<td>0.06</td>
<td>1.11</td>
<td>-0.01</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at August 2015. FTSE Developed, FTSE Developed Minimum Variance, FTSE Developed ERC and FTSE EDHEC-Risk Efficient Developed indexes, September 2003–July 2015. The FTSE Developed Equal Risk Contribution Index and the FTSE Developed Minimum Variance Index are rebalanced semi-annually in March and September. The FTSE EDHEC-Risk Efficient Developed Index is rebalanced quarterly. Factor exposure is the monthly average exposure as detailed in Section 2.1. Past performance is no guarantee of future results. Please see the final page for important legal information.

In Figures 3-6 we show the individual active factor exposures of the three risk-based indexes over time. Risk-based indexes aim to achieve improved risk-adjusted outcomes and/or levels of diversification. The latter objective typically results in consistent positive exposure to size (see Figure 3). However, the value exposure of risk-based indexes varies considerably through time and is currently close to zero (Figure 4). In Figure 5, both the FTSE Developed Minimum Variance and FTSE Developed Equal Risk Contribution indexes exhibit a time varying low volatility tilt, a consequence of aiming to minimise total risk. Conversely, the FTSE EDHEC-Risk Efficient Developed index is orientated towards higher (downside) volatility stocks. This finding is consistent with Blitz (2003). High-quality stocks typically display low levels of volatility and as a consequence both the FTSE Developed Minimum Variance and ERC indexes have (small) positive exposure to quality (Figure 6).
In this section, we have shown that alternatively weighted indexes exhibit significant and variable exposures to common factors such as value, momentum, volatility and size. There is a significant body of work documenting the existence of long-run factor return premia attributable to these risk factors.

Alternatively weighted indexes are commonly combined in order to diversify model or weighting risk, and increasingly to achieve single and multiple factor objectives. In Sections 4 and 5 we show that factor exposure may also be achieved through dedicated factor indexes. Single factor indexes aim to provide exposure to a target risk factor whilst limiting exposure to non-target factors. We illustrate that the factor outcomes associated with single factor indexes are robust compared to an equivalent alternatively weighted index. In Section 5, we propose an alternative approach to constructing multiple factor indexes. We highlight that multiple factor objectives may be achieved more effectively through dedicated factor indexes than combining alternatively weighted indexes which may result in uncontrolled exposure to non-target factors and relatively weak exposure to the factors of interest.

2 For example, Fama & French (1993), Carhart (1997).
4. Factor exposure of factor indexes

4.1 FTSE Factor Indexes

In their book, Active Portfolio Management (1995), Grinold and Kahn show that a factor-mimicking portfolio can be represented by a long-short portfolio. In essence, the long-short portfolio tracks the performance of a specific characteristic (e.g., the value or size effect). More recently, the development of long-only factor indexes has provided tools to assist long-only investors who seek to replicate the performance of factor risk premia directly. In this section, we discuss the characteristics of a set of FTSE factor indexes, which are designed to represent the performance of specific factors.

In Tables 5 and 6 we highlight the exposure of six FTSE single factor indexes to their target factors, as well as to non-target factors. In Table 5 we show the factor exposure of a set of broad factor indexes, derived from the FTSE Developed Index, while in Table 6 we show the results for a set of narrow factor indexes. Narrow factor indexes are derived from the broad indexes by the removal of the least attractive stocks, measured by their contribution to the factor objective, subject to diversification and capacity constraints. For further information on the approach used, please see FTSE (2014a) and FTSE (2014e).

Each index is rebalanced annually in September, with the exception of the Residual Momentum Indexes\(^3\), which are rebalanced semi-annually in September and March. The narrowing process described above is not applied to the Residual Momentum Index, since the speed at which the momentum signal degrades necessitates more frequent rebalancing and retaining the broad approach provides an effective means of limiting the resulting turnover. The Momentum Factor Indexes in Tables 5 and 6 are therefore identical.

<table>
<thead>
<tr>
<th>Index name</th>
<th>Average active exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Illiquidity</td>
</tr>
<tr>
<td>Broad Illiquidity Factor Index</td>
<td>0.91</td>
</tr>
<tr>
<td>Residual Momentum Factor Index</td>
<td>0.01</td>
</tr>
<tr>
<td>Broad Quality Factor Index</td>
<td>-0.04</td>
</tr>
<tr>
<td>Broad Size Factor Index</td>
<td>0.92</td>
</tr>
<tr>
<td>Broad Value Factor Index</td>
<td>0.01</td>
</tr>
<tr>
<td>Broad Volatility Factor Index</td>
<td>-0.08</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at August 2015. Factor exposure relative to the FTSE Developed Index, September 2001 – July 2015. Factor exposure is the monthly average exposure as detailed in Section 2.1. Past performance is no guarantee of future results. Please see the final page for important legal information.

Factor indexes based on a broad universe (Table 5) have exposure to non-target factors that are close to zero, except for size and illiquidity. Both target and non-target factor exposures of the narrow indexes in Table 6 are higher. This is unsurprising, given the narrowing process applied.

\(^3\) A complete definition of Residual Momentum can be found in FTSE (2014d), the FTSEGloba Factor Index Series Ground Rules (FTSE, 2014e) and Appendix E.
Small stocks, for example, tend to be more volatile and less liquid. Consequently, in addition to positive exposure to the target factor (size), the size factor index also exhibits negative exposure to the (low) volatility factor and positive exposure to the illiquidity factor.

While the narrow Value Factor Index exhibits nearly twice the value exposure of the broad Value Factor Index it also displays increased small cap exposure. Fama and French (1992) document a negative correlation between size and value and suggest that cheap stocks tend to be small. Quality stocks tend to be expensive; consequently, the narrow Value Factor Index displays a negative quality tilt. Quality and (low) volatility are positively correlated, resulting in positive exposure of the Volatility Factor Index towards quality and a positive exposure of the Quality Index towards the (low) volatility factor.

### Table 6. Average Active Factor Exposure – Narrow FTSE Developed Factor Indexes

<table>
<thead>
<tr>
<th>Index name</th>
<th>Illiquidity</th>
<th>Residual momentum</th>
<th>Quality</th>
<th>Size</th>
<th>Value</th>
<th>(Low) volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illiquidity Factor Index</td>
<td>1.45</td>
<td>0.05</td>
<td>0.00</td>
<td>1.55</td>
<td>0.01</td>
<td>-0.23</td>
</tr>
<tr>
<td>Residual Momentum Factor Index</td>
<td>0.01</td>
<td>0.44</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>Quality Factor Index</td>
<td>-0.07</td>
<td>-0.01</td>
<td>0.76</td>
<td>-0.09</td>
<td>-0.04</td>
<td>0.18</td>
</tr>
<tr>
<td>Size Factor Index</td>
<td>1.33</td>
<td>0.00</td>
<td>-0.01</td>
<td>1.68</td>
<td>0.04</td>
<td>-0.30</td>
</tr>
<tr>
<td>Value Factor Index</td>
<td>0.19</td>
<td>-0.06</td>
<td>-0.12</td>
<td>0.25</td>
<td>0.70</td>
<td>-0.02</td>
</tr>
<tr>
<td>Volatility Factor Index</td>
<td>-0.19</td>
<td>0.05</td>
<td>0.12</td>
<td>-0.28</td>
<td>0.04</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at August 2015. Factor exposure relative to the FTSE Developed Index, September 2001 and December 2014. Factor exposure is the monthly average exposure as detailed in Section 2.1. Past performance is no guarantee of future results. Please see the final page for important legal information.

Appendix D shows the active factor exposure of the broad and narrow factor indexes constructed from the FTSE Developed index to the target factors through time. The charts confirm that the narrow factor indexes exhibit greater exposure to their target factors than broad indexes through time.

### 4.2 Factor and alternatively weighted indexes

Factor indexes also display relatively high levels of exposure to target factors compared to alternatively weighted indexes. For example, the size factor and the equally weighted index display average size exposure of 1.68 (table 6) and 1.42 (table 2) respectively; the comparable value exposure for the Value Factor Index and the Fundamental Index are 0.7 (table 6) and 0.32 (table 3); the Volatility Factor Index and the Minimum Variance Index exposures to volatility are 0.58 (table 6) and 0.23 (table 4). Figures 7-9 suggest that the factor indexes exhibit greater exposure at any point in time to the target factor than alternatively weighted indexes that are commonly used to gain exposure to the same factor.

However, we express some reservations regarding this conclusion, since common factor definitions are used to both create and evaluate the factor exposure of the factor indexes. The alternatively weighted indexes are constructed using variants of these definitions. Consequently, our results are biased in favour of a conclusion that factor indexes exhibit greater factor exposure than alternatively weighted indexes. We address this concern in the next section.
4.3 Confirmation of factor exposures using alternative factor definitions

In order to address the reservations expressed in Section 4.2, we re-evaluate the factor exposures of both factor and alternatively weighted indexes using the characteristics used to form the alternatively weighted indexes. In Section 4.2, the factor exposure of alternatively weighted indexes to each factor is calculated using the factor definitions used in the construction of the FTSE Global Factor Index Series. A valid criticism of this approach is that the factor exposure of the factor indexes is therefore upwardly biased: for example, a value factor index
formed using the book-to-price ratio as the measure of value is likely to exhibit higher value factor exposure when measured using book-to-price than a fundamental index constructed using some other value metric (e.g. cash flow).

We restrict our assessment to the volatility and value exposures of the FTSE Developed Minimum Variance and FTSE RAFI Developed 1000 indexes and Low Volatility and Value Factor indexes. We do not re-examine the levels of size exposure displayed by the Equally Weighted and Size Factor indexes as market capitalisation represents a fair metric for comparison.

The FTSE Developed Minimum Variance Index utilises two-years of daily USD returns in the construction of the covariance matrix. Therefore, we re-examine the exposure of the FTSE Developed Volatility Index and the FTSE Developed Minimum Variance Index to a volatility measure constructed using two-years of daily total USD returns.

The FTSE RAFI Developed 1000 Index is constructed from a combination of book value, cash flow, dividends and sales. We therefore examine the exposure of the Value Factor Index and the FTSE RAFI Developed 1000 index to a simple equally weighted composite of four value metrics book-to-price, cash flow yield, dividend yield and sales to price. Note that the FTSE Value Factor Index is an equally weighted composite of earnings yield, cash-flow yield and sales to price, the latter measured relative to the country median.

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**Figure 10. Active Volatility Exposure – FTSE Developed Minimum Variance Index and FTSE Developed Volatility Factor Index**

**Figure 11. Active Value Exposure – FTSE RAFI Developed 1000 and FTSE Developed Value Factor Index**

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Figures 10 and 11 indicate the factor exposures of dedicated factor indexes are robust to the change in factor definitions, displaying persistently greater exposure to the factor of interest than the comparable alternatively weighted index. However, it is important to recall that alternatively weighted indexes do not explicitly target factor exposure, but a range of other objectives. For example, FTSE Developed Minimum Variance Index expressly targets reductions in index level volatility; the FTSE RAFI Developed 1000 has a dynamic value objective, but also explicitly addresses capacity and implementation considerations, employing five year averages in the construction of each metric (apart from book value), in order to limit turnover.
4.4 Factor exposure decay

Typically, the strength of a factor index (i.e. its exposure to the target factor) begins to decay after the index is rebalanced. This decay is evident in the jagged exposure patterns in Figures 12-13 and in Figures D1-D6 of Appendix D.

Two distinct effects are evident; one the direct result of genuine factor exposure decay and one an artefact of the factor index construction methodology. The former effect is illustrated in Figure 13, where the exposure of the Momentum factor index incorporating a semi-annual rebalance is shown. A policy of regular index rebalancing is therefore necessary to maintain factor exposure. The rebalancing frequency recognises the speed of decay in momentum exposure. The signal decay for the Size, (Low) Volatility and Illiquidity factors is less pronounced (see Figures D1, D4 and D5 in Appendix D).

The second effect is seen in factor indexes incorporating financial statement information. An examination of the Quality factor index exposure in Figure 12 reveals a sharp decline in factor exposure immediately prior to each index rebalancing. This arises as a consequence of imposing a six month lag on the use of all accounting data in the creation of the index history in an attempt to mitigate look-ahead bias.

The majority of index constituents have a December financial year end and the effect of the data lag is that new accounting information is available in the following July and incorporated in the index in September at the next index rebalance. Factor exposure however, is calculated each month and newly available accounting information is typically available in July. The decline in factor exposure observed prior to each rebalancing and the subsequent increase at the September rebalance is a consequence of this timing mismatch. The effect is less pronounced in any live index as new information arrives in a more granular fashion throughout the year and is reflected immediately in exposures as no lag is required.

Source: FTSE Russell. Data as at August 2015. FTSE Developed Quality Factor Index and FTSE Developed Residual Momentum Factor Index September 2001 – July 2015. Past performance is no guarantee of future results. Please see the final page for important legal information.
5. Multi-factor indexes

5.1 Combining factors

The pay-off to factor exposure is widely considered compensation for bearing risk associated with the factor (a concept inherent in the term “factor risk premium”). Factor risk premia are variable: they accrue over time, but not all of the time. Individual factors also follow different performance regimes: value, for example, is typically considered to exhibit a pro-cyclical performance, performing strongly during periods of strong economic growth and higher risk appetite. In contrast, the performance of quality is typically countercyclical. In light of the difficulty of successfully timing or employing a factor rotation strategy, diversified factor combinations are increasingly employed. Thus, multi-factor indexes attempt to limit the risk, magnitude and extent of periodic index underperformance and to offer potential improvements in risk-adjusted outcomes, compared with exposure to single factors.

Much of the discussion concerning factor combinations is couched in terms of reductions in tracking error and/or turnover, rather than exposure to the set of target factors. However, a reduction in tracking error arising from a reduction in exposure to the factors of interest as a result of combining factors is not surprising if the factors are a meaningful determinant of performance. Consequently, we would argue that a sensible objective of any factor index methodology is to maintain comparable levels of factor exposure to those achieved in a single factor context.

Two common approaches to achieving multiple factor outcomes are to combine either multiple single factor indexes to create a composite index or to combine multiple security-level factor scores to create a composite factor index. Where factors are positively correlated, combining factors (via composite indexes or an index of a composite factor) may be a sensible approach, but any resulting reduction in tracking error or turnover is likely to be small. Combining negatively correlated factors is likely to yield a greater reduction in tracking error and turnover, but these effects cannot be viewed independently of the effects on factor exposure that result when combining factors. In practice, turnover and tracking error reductions are often the result of the netting off of individual factor index weights and result in a muted exposure to all target factors. Netting off exposure to target factors by combining negatively correlated factors thus defeats original goal of achieving simultaneous exposure to multiple factors. This may not be a desirable outcome and a general approach to achieving controlled multiple factor exposures is therefore required.

In practice, alternatively weighted indexes are frequently combined to achieve factor objectives. For example quality, value and volatility factor exposures may be obtained by averaging the weights of a quality factor index, a fundamental index (e.g. FTSE RAFI Developed 1000 Index) and a low volatility index (e.g. FTSE Developed Minimum Variance Index).

Such an approach to achieving multiple factor objectives lacks generality and requires a variety of ad hoc weighting schemes. Factor outcomes are typically incidental to these approaches and moreover the findings in Sections 3.2 and 3.3 demonstrated the time-varying nature of target factor exposures and the substantial non-target exposures displayed by alternatively weighted indexes. This in conjunction with the off-setting effects discussed above, suggest that attempting to achieve multiple factor objectives through combinations of alternatively weighted indexes can be improved upon.
In Section 1.2, we highlighted that multiple factor objectives can be achieved by the repeated application of a factor tilting mechanism to different factors i.e. by first tilting index weights towards the first factor of interest and tilting the result towards a second factor of interest. In Sections 5.2 and 5.3 we illustrate this and alternative approaches, assessing the factor exposures of each.

### 5.2 A Composite of single factor indexes verses multiple tilts

In Figure 14, below, we illustrate the factor exposures that result from two approaches to combining factors in order to achieve simultaneous exposure to value, quality and volatility: first, from a multiple tilt perspective and second via a composite index using the average weight of stocks in component single factor indexes (security weights in the composite index are the average of the weights in the FTSE Developed Value, Quality and Volatility Factor Indexes). Both the multiple tilt and composite factor indexes are rebalanced annually in September.

The multiple tilt approach first applies a tilt towards the value factor and then another towards a composite of the quality and volatility factors. A composite quality (low) volatility factor is used, since these two factors are typically positively correlated (high quality stocks tend to exhibit low volatility).

The results shown in Figure 14 are telling. The index created using the sequential tilting approach exhibits significant exposure to the value, volatility and quality factors that are broadly comparable to the target exposure of each single factor index. However, the average weighting approach exhibits relatively weak exposure to all the factors of interest.

Figure 14. FTSE Developed active factor exposure: Single factor indexes and multiple tilt approach

Source: FTSE Russell. Data as at August 2015. Averages taken between September 2001 and July 2015. Factor exposure is the monthly average exposure as detailed in Section 2.1. Past performance is no guarantee of future results. Please see the final page for important legal information.
5.3 A Composite of alternatively weighted indexes versus multiple tilts

In a similar vein, we target the same multiple factor objective (quality, value and volatility) by combining the security weights of each alternatively weighted index in a composite index. The average weights of the FTSE Developed Quality Factor Index, the FTSE RAFI Developed 1000 Index and the FTSE Developed Minimum Variance Index form the composite index.

The rebalance frequency and timing of the three component indexes are different; the FTSE Developed Quality Index rebalances annually in September; the FTSE RAFI Developed 1000 Index rebalances annually in March and the FTSE Developed Minimum Variance Index rebalances semi-annually in March and September. To provide a fairer comparison, the composite index is rebalanced twice a year in March and September using the prevailing weights of each component index.

The factor exposures of each approach to the metric(s) used to form the factor indexes are illustrated in Figure 15. For comparison, we include the multiple tilt outcomes from Figure 14, calculated over the March 2007-July 2015 period. The results confirm the findings in the previous section. The average weighting approach displays relatively weak levels of exposure to all the target factors. Furthermore, the composite approach inherits the unintended factor exposure to a non-target factor (size) from the FTSE Developed Minimum Variance Index.

Indeed this size exposure is actually larger than the exposures to the target factors. A multi-factor index created by sequentially tilting exhibits significant exposure to all factors of interest.

Figure 15. Active factor exposure: Composite of alternatively weighted indexes and multiple tilt approach
In Figure 16, we compare the factor exposures of a composite index of single factor indexes (from Figure 14) with those of the composite index of alternatively weighted indexes (from Figure 15) and the multiple tilt approach. The multiple-tilt approach exhibits significant exposure to value, quality and volatility while maintaining low levels of exposure to size and momentum. The composite indexes exhibit low levels of exposure to the target factors and in addition, a composite of alternatively weighted indexes display limited control over non-target exposures. The non-target factor outcome is driven by the magnitude of the factor exposures embedded in the underlying component indexes.

**Figure 16. Active factor exposure: Composite of single factor indexes, composite of alternatively weighted indexes and multiple tilt approaches**

![Graph showing factor exposures](image)

Source: FTSE Russell. Data as at August 2015. Averages taken between March 2007 and July 2015. Factor exposure is the monthly average exposure as detailed in Section 2.1. Past performance is no guarantee of future results. Please see the final page for important legal information.

The results in section 5.2 and 5.3 highlight that an average weighting or composite index approach to combining factors may not be optimal, particularly when factors are negatively correlated, as is the case with value and quality. In this situation combining single factor indexes or alternatively weighted indexes results in relatively weak exposure to all factors of interest compared to the sequential tilt approach.

The sequential tilt approach is similar to double sorting. For example, consider value and size in Figure 17; the double sort approach separates the universe by market capitalisation into small, medium and large stocks. Each size cohort, containing an equal proportion of the total market capitalisation (and therefore a different number of stocks), is then divided into three groups by value. Typically, the most attractive group (from the perspective of factor risk premia) is positioned in the bottom right corner while the least attractive group is positioned in the top left corner. In Figure 18, the small value group (in yellow) has historically outperformed the big growth group (in blue), realising both the size and value effects.
A double sort approach is commonly used to assess the efficacy of factor signals. However, a resulting portfolio may include both long and short (negative) positions, may exhibit relatively high turnover and may leave constituent weights undefined, rendering the index impractical for use in an investment context. Figure 18 illustrates the sequential tilt approach. Similar to Figure 17, yellow/orange represents the most attractive and blue the least attractive category. As one moves from the bottom right to the top left, the stock characteristics gradually transition from small value to big growth. The tilt approach is long–only, with a positive weight assigned to all constituents. The increased granularity contributes limits turnover, resulting in a more practical index. Furthermore, the results of the double sort depend on the order in which the sorts are applied, whilst the ordering of the sequential tilts does not affect outcomes.

**Figure 17. Double Sort Approach**

<table>
<thead>
<tr>
<th>Big Growth</th>
<th>Big Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Growth</td>
<td>Small Value</td>
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</table>

**Figure 18. Sequential Tilt Approach**

Source: FTSE Russell.

### 5.4 A holistic approach to factors

The foregoing highlights that multiple factor objectives must be viewed holistically. A factor index construction methodology should target multiple outcomes simultaneously, rather than approaching each individual factor component separately. We have shown that alternatively weighted indexes exhibit factor characteristics. However, a simple average of these indexes results in weaker exposure across target factors. From an investor perspective, simply averaging indexes is tantamount to a preference for weaker exposure to all target factors.

The importance of efficient factor capture becomes apparent when one considers that factor preferences and objectives may be subject to change. Consider a policy change resulting in a desire to add a quality dimension to an existing value objective, possibly prompted by a view on a decline in risk appetite. Simply combining an existing value factor index (or fundamental index) with a quality factor index (i.e., reallocating a portion of the portfolio’s value allocation to quality) is likely to result in limited exposure to both factor objectives. If such a forecast of the market’s risk appetite is correct, the weak levels of quality exposure will provide limited downside protection. Conversely, a return to more normal levels of risk appetite will meet with limited reward as a consequence of the limited value exposure.
The sequential tilt approach provides a possible solution. In Section 1.2, we highlighted that a factor tilt can be applied from any starting point, not just the capitalisation weighted index. For example, a quality tilt may be applied to an existing fundamental index or to a value factor index to achieve the required quality factor overlay. The difference between the starting index and the tilted index (the delta or completion portfolio) contains the information required to overlay or achieve the new factor objective.

Figure 19 shows the benefit of this operation. Suppose an investor runs a fundamentally weighted fund, tracking the FTSE RAFI Developed 1000 Index. The initial value exposure of the FTSE RAFI Developed 1000 Index is around 0.3 (standard deviations above the average stock). A composite of the FTSE RAFI Developed 1000 and a quality index results in a comparable improvement in quality exposure to that achieved by the tilt approach. However, the red bar in Figure 19 highlights that tilt approach additionally maintains the existing levels of value exposure.

To achieve such outcome in practice, a fund need only to add an allocation to quality, i.e., overlay the difference or completion portfolio that captures the differences between the initial value fund and the tilted index.
Conclusion

Alternatively weighted indexes were originally developed to address the concentration risks in capitalisation weighted indexes. Many alternatively weighted indexes also offer incidental exposure to risk factors that have historically been associated with positive long-run risk premia. Factor indexes are designed to provide explicit exposure to these risk premia. FTSE’s analysis suggests that factor indexes provide more controlled exposure to factor risk premia than alternatively weighted indexes. We also highlight the importance of the mechanism used to combine factors to achieve multiple factor objectives. A composite of single factor indexes or of alternatively weighted indexes (or an index based on a composite factor) is likely to result in relatively weak levels of factor exposure (compared to individual single factor indexes), particularly when factors are negatively correlated. A sequential tilting approach provides an effective and general means of achieving multiple factor objectives and a natural mechanism for identifying the completion portfolio to encompass changing factor objectives.
References


Appendix A: FTSE Developed Equally Weighted Index active factor exposures

Figure A1. Active Value Factor Exposure

Figure A2. Active Size Factor Exposure

Figure A3. Active Quality Factor Exposure

Figure A4. Active Volatility Factor Exposure

Figure A5. Active 12-Month Momentum Factor Exposure

Figure A6. Active Illiquidity Factor Exposure

Source: FTSE Russell. Data as at August 2015. Average taken between September 2001 and July 2015. Monthly exposure is calculated as the index exposure relative to the capitalisation weighted FTSE Developed Index. The methodology follows that outlined in Section 2.1. Returns shown may reflect hypothetical historical performance. Past performance is no guarantee of future results. Please see the final page for important legal information.
Appendix B: FTSE RAFI Developed 1000 Index Active Factor Exposures

Figure B1. Active Value Factor Exposure

Figure B2. Active Size Factor Exposure

Figure B3. Active Quality Factor Exposure

Figure B4. Active Volatility Factor Exposure

Figure B5. Active 12-Month Momentum Factor Exposure

Figure B6. Active Illiquidity Factor Exposure

Source: FTSE Russell. Data as at August 2015. Average taken between September 2001 and July 2015. Monthly exposure is calculated as the index exposure relative to the capitalisation weighted FTSE Developed Index. The methodology follows that outlined in Section 2.1. Returns shown may reflect hypothetical historical performance. Past performance is no guarantee of future results. Please see the final page for important legal information.
Appendix C: FTSE developed risk based indexes active factor exposures

Figure C1. 2 Active Value Factor Exposure

Figure C2. Active Size Factor Exposure

Figure C3. Active Quality Factor Exposure

Figure C4. Active Volatility Factor Exposure

Figure C5. Active 12-Momentum Factor Exposure

Figure C6. Active Illiquidity Factor Exposure

Source: FTSE Russell. Data as at August 2015. FTSE Developed Minimum Variance, FTSE Developed ERC and FTSE EDHEC-Risk Efficient Developed indexes, September 2003–July 2015. Monthly exposure is calculated as the index exposure relative to the capitalisation weighted FTSE Developed Index. The methodology follows that outlined in Section 2.1. Returns shown may reflect hypothetical historical performance. Past performance is no guarantee of future results. Please see the final page for important legal information.
Appendix D: Active factor exposure of broad and narrow factor indexes

Figure D1. Active Value Factor Exposure

Figure D2. Active Size Factor Exposure

Figure D3. Active Quality Factor Exposure

Figure D4. Active Volatility Factor Exposure

Figure D5. Active Illiquidity Factor Exposure

Figure D6. Active Residual Momentum Factor Exposure

Source: FTSE Russell. Data as at August 2015. Averages taken between September 2001 and July 2015. The underlying universe is FTSE Developed. Monthly exposure is calculated as the index exposure relative to the capitalisation weighted FTSE Developed Index. The methodology follows that outlined in Section 2.1. Returns shown may reflect hypothetical historical performance. Past performance is no guarantee of future results. Please see the final page for important legal information.
Appendix E: Factor definitions

E1. Illiquidity

Log Amihud Ratio. The Amihud Ratio is the price impact per dollar traded; the greater the Amihud Ratio, the greater the degree of illiquidity.

\[
\text{Amihud Ratio}_t = \text{Median} \left( \frac{|R_{it}|}{\text{Traded Value}_{it}} \right)
\]

Where \(R_{it}\) is the USD total return of stock \(i\) on day \(t\); \(\text{Traded Value}_{it}\) is the daily trading value of stock \(i\) measured in USD. The median daily Amihud Ratio is determined over the 12 month period prior to the review month.

E2. 12-Month Momentum

The cumulative total local return over the last 12 months excluding the last month.

E3. Quality

Quality is a composite of Profitability and Leverage measures. Three individual Profitability measures and a single measure of Leverage are considered.

Profitability

Profitability consists of an average of the individually normalised values of the following three measures. All profitability measures are calculated relative to the relevant regional median stock level.

a) \(\text{ROA} = \frac{\text{Net Income}}{\text{Average Total Assets}}\)

b) \(\Delta \text{Asset Turnover} = \frac{\text{Sales}_t}{\text{Total Assets}_t} - \frac{\text{Sales}_{t-1}}{\text{Total Assets}_{t-1}}\)

c) \(\text{Accruals} = \frac{\text{WC} + \text{NCO} + \text{FIN}}{\text{Average Total Assets}} * -1\)

Where:

Average Total Assets = \((\text{Total Assets}_t + \text{Total Assets}_{t-1})/2\)

\(\text{WC} \) (Working Capital) = \((\text{Current Assets} - \text{Cash & Short-term Investments}) - (\text{Current Liability} - \text{Short-term Debt})\)

Non-current Net Operating Assets (NCO) = \((\text{Total Assets} - \text{Current Assets} - \text{Investments and Advances}) - (\text{Total Liability} - \text{Current Liabilities} - \text{Long-term Debt})\)

Net Financial Assets (FIN) = \((\text{Short-term Investments} + \text{Long-term Investments}) - (\text{Long-term Debt} + \text{Short-term Debt} + \text{Preferred Stock})\)

Leverage Ratio

Leverage is the ratio of Operating Cash Flow to Total Debt measured relative to the regional industry (ICB) median stock level.

\[
\text{Leverage Ratio} = \frac{\text{Operating Cash Flow}}{\text{Debt}}
\]
E4. Size
Log Full Market Capitalization.

E5. Value
Value is represented by a composite of three common valuation measures:

a) Cash-flow Yield = Latest Annual Cash-Flow / Full Market Capitalisation
b) Earnings Yield = Latest Annual Net Income / Full Market Capitalisation
c) Sales to Price = Latest Annual Sales / Full Market Capitalisation

Sales to Price is calculated in relative to the country median level.

E6. Volatility
Volatility is defined as the standard deviation of five years of weekly (Wednesday to Wednesday) total local returns prior to the rebalance month.

E7. Residual Momentum
Past returns are decomposed into stock specific, systematic and residual components.

$$R_{it} = \alpha_i + \sum_k \beta_{ik} F_{kt} + \varepsilon_{it}$$

Where $R_{it}$ is the local total return of stock $i$ in period $t$; $\alpha_i$ the stock specific return; $\beta_{ik}$ the stock exposure to risk factor $k$; $F_{kt}$ the return to risk factor $k$ in period $t$, and $\varepsilon_{it}$ the residual return. A two factor risk model is employed, where the risk factors are the relevant country and universe industry total returns in the local currency of stock $i$.

Residual Momentum is defined as the Residual Sharpe Ratio, which is calculated by estimating the equation above on a rolling monthly basis using total local monthly returns over a 36 month period. The equation is re-estimated for each of the eleven months in the twelve month period prior to the review month, excluding the most recent month.

For each of the eleven monthly regressions, the average residual return over the most recent twelve months is calculated. The Residual Sharpe Ratio is derived as the ratio of the mean and standard deviation of the eleven month time-series of average residual returns.
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