Factor exposure indexes
Value factor

1. Summary

The value effect is one of the most studied market anomalies [2-5]. The value effect or value premium refers to the tendency of stocks with lower valuation ratios to earn above average returns over the long run. For example, the cross-sectional variation of stock returns across countries can be partly explained by a global value factor [6]. Such a value effect has been observed across many different markets, regions and sample periods [1].

In this paper, we use a combination of several common valuation measures to capture the value premium. We assess forecast measures of Earnings Yield, Book to Price, Cash Flow Yield, Sales to Price and Dividend Yield. All valuation measures are calculated on a 12-month forward basis using IBES estimates. We find a value premium that is economically significant using both absolute and relative measures of value. However, the ability to capture any value premium varies across valuation measures.

In terms of absolute measures of value, Earnings Yield is the most significant, followed by Cash Flow Yield, Sales to Price, Book to Price and finally Dividend Yield. Country relative measures of value are generally comparable to absolute measures of value in their ability to identify a value premium: the exceptions are Earnings Yield which is poorer and Sales to Price, which is superior to its absolute measure. Furthermore, when value is measured by Earnings Yield, Cash Flow Yield and Sales to Price, value effects are relatively persistent over time.

The strength and persistency of individual value effects and levels of correlation amongst individual valuation measures suggest that a combination of Earnings Yield, Cash Flow Yield and Sales to Price is appropriate. We illustrate the simulated historical performance characteristics of various composite value factors within the context of a set of illustrative indexes. Given the high correlation between Cash Flow Yield and Book to Price and the similarity of index level characteristics between indexes formed on two and three-factor composites, we prefer a three-factor composite value measure. Since our analysis is very much based "in sample", this is a cautious choice which ensures greater factor diversity. This choice has the further advantage that our composite factor has a greater orientation towards factors that are less susceptible to manipulation (Cash Flow Yield and Sales to Price) than to those that are (Earnings Yield).

Index characteristics, including capacity, levels of diversification and the degree of exposure to the factor of interest are robust to the frequency and timing of the index rebalance. Rebalance frequency has a substantial impact on turnover. An annual rebalance minimises turnover; we make the pragmatic decision to rebalance annually in September order to synchronise index rebalances with reviews of the underlying index universe and other factor (Quality) indexes.

We construct a broad value index that includes all underlying index constituents. We subsequently form a narrow value index containing fewer stocks and displaying greater exposure to value and assess the robustness of performance outcomes. This increased exposure to value results in improvements in performance compared to a broad value index.

The paper is structured as follows: Section 2 assesses alternative measures of valuation from the perspective of signal strength, persistency and correlation.

Section 3 examines practical factor indexes based on individual valuation measures. A broad composite value index is examined in Section 4 in the context of an appropriate rebalance frequency. Section 5 examines the implications of forming a narrow composite value index. Section 6 concludes.

2. Regression analysis

In order to assess the pure factor performance characteristics of each value factor, we perform the regression analysis detailed in Appendix A. This analysis results in the return to each ideal factor mimicking portfolio and therefore avoids any dependence on the method of index construction chosen.

We examine constituents of the FTSE Developed universe, performing monthly cross-sectional regressions for the period March 2000 to December 2013. Table 1 shows the average ideal monthly factor returns and t-stats for each individual factor with and without the inclusion of industry and country membership as additional factors. An equally weighted regression scheme and monthly local price returns are used to form each cross-sectional regression.

Earnings Yield is the strongest and most significant factor in absolute terms, followed by Cash Flow Yield, Sales to Price, Book to Price and finally Dividend Yield. All industry relative measures result in lower absolute levels of significance and mean returns. All country relative measure results are comparable to the absolute factor outcomes with the exceptions of Earnings Yield which is poorer and Sales to Price which is superior to its absolute measure. This suggests that absolute value measures may be used for Earnings Yield, Book to Price, Cash Flow Yield and Dividend Yield, whilst a country relative measure is appropriate for Sales to Price. Throughout the rest of this document we adopt these measures.

Chart 1 displays the two year rolling average t-stats through time for each valuation factor. The graphic illustrates the significance of Earnings Yield, Cash Flow Yield and Sales to Price is relatively persistent through time. Although Book to Price is relatively stable, post early 2007 t-stats are mainly negative. Dividend yield is relatively unstable and insignificant throughout most of the period.

Table 1. Factor Returns and t-stats

| Factor	Absolute		Relative to Industry		Relative to Country	
	Return	t-Stat	Return	t-Stat	Return	t-Stat
Earnings Yield	0.24%	1.10	0.20%	0.89	0.21%	0.89
Book to Price	0.16%	0.62	0.14%	0.56	0.18%	0.66
Cash Flow Yield	0.22%	0.89	0.21%	0.77	0.22%	0.86
Sales to Price	0.19%	0.75	0.14%	0.50	0.24%	0.93
Dividend Yield	0.11%	0.34	0.08%	0.22	0.13%	0.37

Source FTSE: FTSE Developed, mean, monthly factor return and cross-sectional t-statistic; March 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

Chart 1. Persistence of Significance
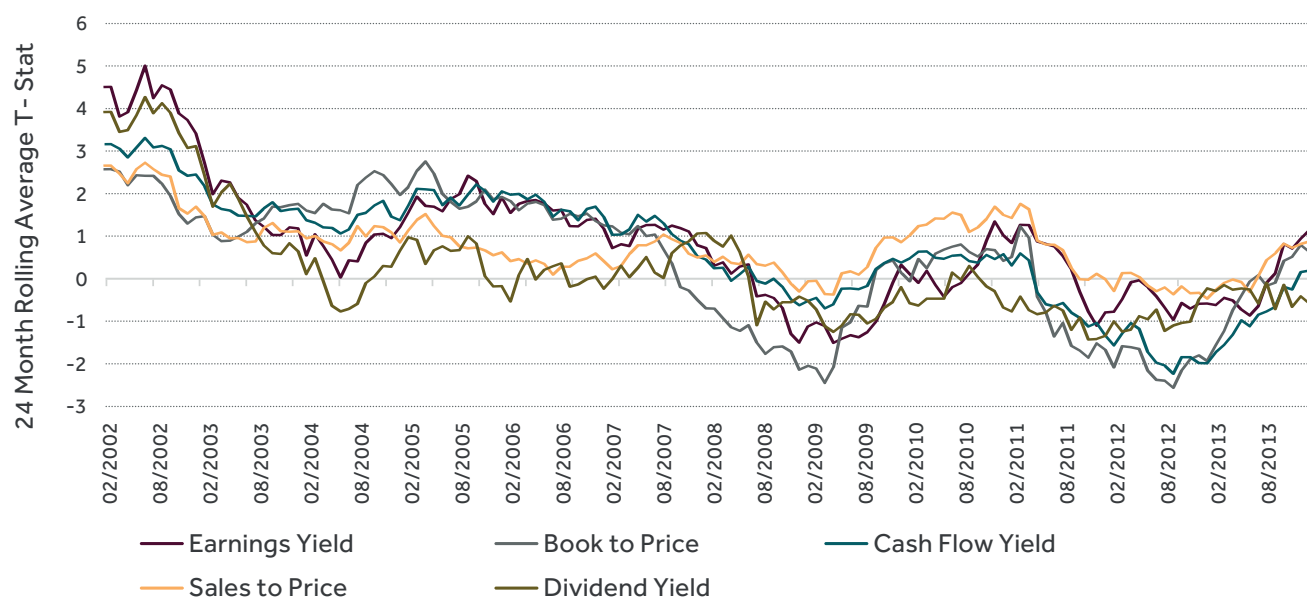
Source FTSE: FTSE Developed, rolling two year average t-statistic; March 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

Chart 2 shows the cumulative ideal factor return for each value factor. The performance of value in general has been flat post September 2007. However all factors apart from Dividend Yield capture the "value rally" post the September 2008 global financial crisis.

Chart 2.Cumulative Factor Returns
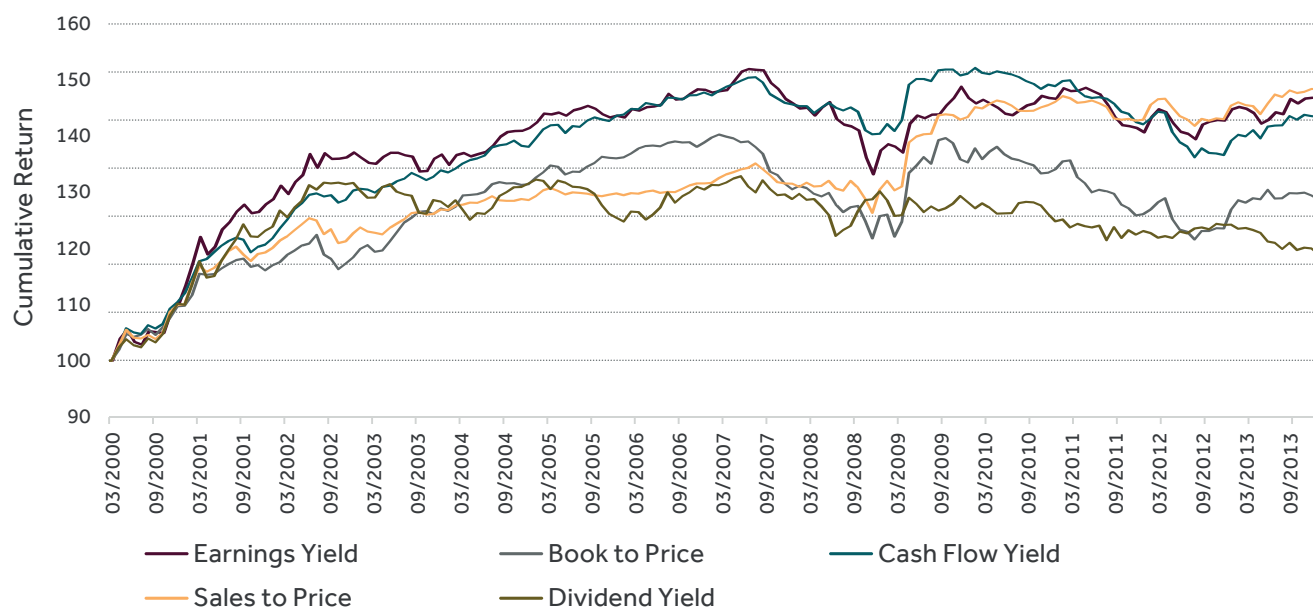
Source FTSE: FTSE Developed; March 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

Table 2 shows the correlation between monthly ideal factor returns. Note that Book to Price, Cash Flow Yield and Sales to Price are all strongly positively correlated to one another. Earnings Yield shows a weaker positive correlation with all factors, whilst Dividend Yield is only notably positively correlated with Earnings Yield.

Table 2. Correlation of Factor Returns

	Earnings Yield	Book to Price	Cash Flow Yield	Sales to Price	Dividend Yield
Earnings Yield	1.00	0.48	0.68	0.66	0.58
Book to Price	–	1.00	0.81	0.80	0.22
Cash Flow Yield	–	–	1.00	0.81	0.31
Sales to Price	–	–	–	1.00	0.29
Dividend Yield	–	–	–	–	1.00

Source FTSE: FTSE Developed, monthly factor return correlation; March 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

Table 3 shows the cross-sectional correlation of raw value factor characteristics i.e. between the valuation measures themselves rather than between the factor returns of each valuation measure. This pattern is weaker, but similar to that observed for factor returns.

Table 3. Correlation of Factor Characteristics

	Earnings Yield	Book to Price	Cash Flow Yield	Sales to Price	Dividend Yield
Earnings Yield	1.00	0.26	0.48	0.25	0.38
Book to Price	–	1.00	0.46	0.29	0.14
Cash Flow Yield	–	–	1.00	0.45	0.21
Sales to Price	–	–	–	1.00	0.05
Dividend Yield	–	–	–	–	1.00

Source FTSE: FTSE Developed, mean monthly cross-sectional factor characteristic correlation; March 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

3. Single factor indexes

In this section we create factor indexes premised on individual value factor characteristics. We employ the index construction techniques set out in 'Factor Exposure Indexes – Index Construction Methodology', FTSE (2014) [7]. Briefly, we map a normalised valuation measure (Z-Score) to a score in the range of zero to one using a cumulative normal mapping. This score is combined with a stock's weighting in an underlying index to determine individual stock weightings in the value factor index. The approach is tantamount to a factor tilt on an underlying index, where the strength (and number) of any tilts and the degree of diversification may be controlled.

Simulated performance figures in the remainder of this document are USD price returns for the period May 2000 to December 2013, where a monthly rebalance frequency is utilised for illustrative purposes only.

Table 4 shows the simulated historical performance figures of indexes that result from tilting away from an underlying market capitalisation weighted index towards each of the alternative value measures. We also show the resulting index level factor exposure (the weighted sum of individual stock value factor Z-Scores, see Appendix B) relative to the underlying universe and the degree to which each index loads on the ideal factor index (see appendix A for details of this calculation). Finally, we assess index capacity as measured by the weighted average capacity ratio (WCR) and consider the degree of diversification via the average Effective N through time (see Appendix B for both definitions).

Table 4. Performance of Value Factor Indexes: FTSE Developed

	FTSE Developed	Earnings Yield	Book to Price	Cash Flow Yield	Sales to Price	Dividend Yield
Geometric Mean (%)	1.89	3.50	2.31	3.17	3.58	2.81
Volatility (%)	17.06	17.94	18.13	17.72	17.38	17.20
Volatility Reduction (%)		-5.17	-6.32	-3.90	-1.91	-0.84
Sharpe Ratio	0.11	0.20	0.13	0.18	0.21	0.16
DD (%)	-58.95	-61.84	-63.87	-60.89	-59.52	-61.94
Two Way Turnover (%)		105.26	101.63	108.19	76.42	69.13
Excess (%)		1.58	0.41	1.26	1.67	0.90
Tracking Error (%)		2.67	3.09	2.44	1.65	3.18
Information Ratio		0.59	0.13	0.52	1.01	0.28
Alpha (%)		1.59	0.44	1.27	1.66	0.95
Alpha T-Stat		2.32	0.55	1.99	3.82	1.12
Beta		1.04	1.05	1.03	1.01	0.99
Number of Stocks	1941	1941	1941	1941	1941	1941
Effective N	340	275	350	314	309	265
Capacity (WCR)	1.00	1.29	1.45	1.32	1.39	1.31
Rel. Factor Exposure		0.50	0.57	0.53	0.50	0.52
Factor Loading	0.00	0.60	0.68	0.58	0.65	0.53

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

Earnings Yield, Cash Flow Yield and Sales to Price have historically proven superior measures of value to Book to Price and Dividend Yield on most performance measures. However, a Book to Price value index is the most diversified (measured by Effective N) and the Dividend Yield value index exhibits substantially lower levels of two way turnover.

All tilted indexes achieve a similarly high loading on the relevant ideal factor index and exhibit substantial exposure to the factor of interest relative to the underlying index.

Chart 3 shows the simulated historical cumulative performance for each factor index relative to FTSE Developed through time.

Table 5 shows the simulated performance figures for a corresponding set of value indexes, where the factor tilt is applied to an equally weighted version of the FTSE Developed index.

As expected, the capacity of these indexes is lower than the market capitalisation tilted counterparts, whilst levels of diversification (Effective N) are superior. Each index continues to exhibit a high loading on the relevant ideal factor index and a superior factor exposure relative to the underlying index.

The relative performance of both sets of indexes is broadly similar. Relative performance is the result of the degree of factor exposure rather than underlying index weighting scheme.

Chart 3. Cumulative Relative Performance of Value Factor Indexes: FTSE Developed
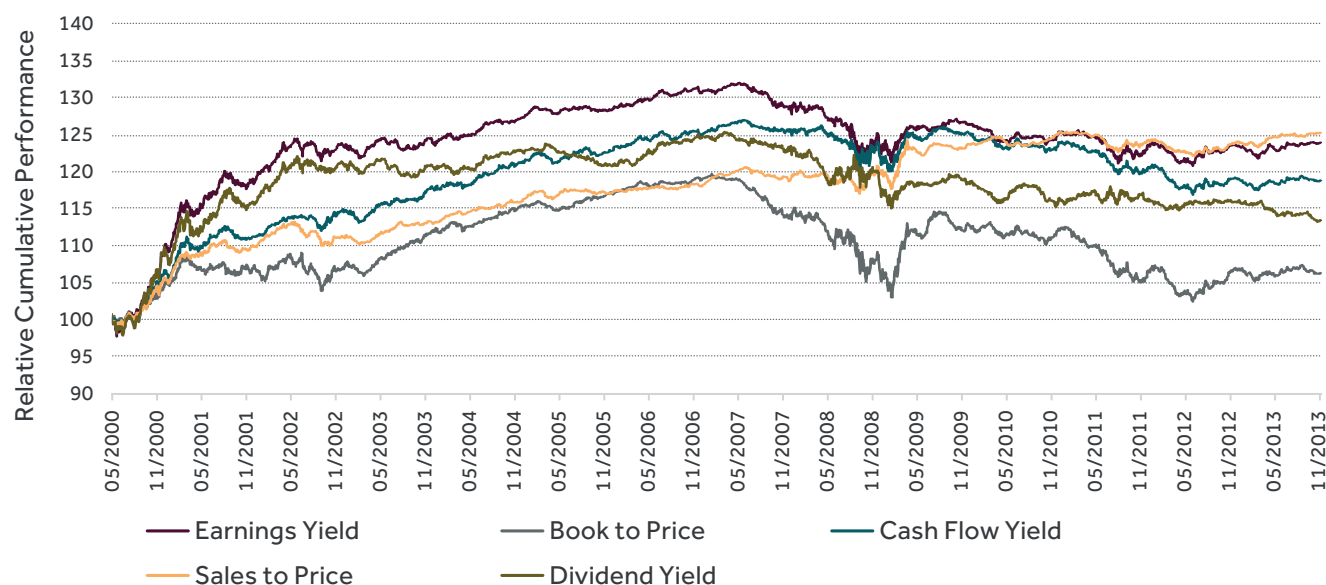
Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

Table 5. Performance of Value Factor Indexes: FTSE Developed Equally Weighted

	FTSE Developed	Earnings Yield	Book to Price	Cash Flow Yield	Sales to Price	Dividend Yield
Geometric Mean (%)	6.16	7.78	7.18	7.70	7.68	7.26
Volatility (%)	15.83	16.65	16.51	16.38	16.30	16.10
Volatility Reduction (%)		-5.21	-4.31	-3.53	-3.00	-1.75
Sharpe Ratio	0.39	0.47	0.43	0.47	0.47	0.45
DD (%)	-58.56	-63.06	-61.18	-60.21	-59.71	-61.66
Two Way Turnover (%)	95.80	177.79	154.86	166.59	139.28	136.56
Excess (%)		1.53	0.96	1.46	1.44	1.04
Tracking Error (%)		2.48	2.29	1.72	1.32	2.65
Information Ratio		0.62	0.42	0.84	1.09	0.39
Alpha (%)		1.36	0.83	1.32	1.31	1.05
Alpha T-Stat		2.12	1.39	2.98	3.94	1.48

	FTSE Developed	Earnings Yield	Book to Price	Cash Flow Yield	Sales to Price	Dividend Yield
Beta		1.04	1.03	1.03	1.03	1.00
Number of Stocks	1941	1941	1941	1941	1941	1941
Effective N	1941	1370	1292	1162	1313	1310
Capacity (WCR)	6.90	10.04	13.83	12.66	11.75	11.04
Rel. Factor Exposure		0.79	0.94	1.26	0.82	0.87
Factor Loading	0.00	0.63	0.62	0.61	0.61	0.56

Source FTSE: FTSE Developed; equally weighted underlying index; USD price returns; May 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

4. Composite value indexes

In this section, we create composite value factor indexes incorporating two, three, four and five of the individual value measures under consideration. We consider only tilts away from a market capitalisation weighted underlying index. Our results are robust to the use of an equally weighted underlying index. For each composite value index, factors are selected on the basis of the most significant t-stat values in Table 2.

All individual valuation measures are positively correlated (See Tables 3 and 4). Consequently, we sum individual cross-sectional Z-Score measures of value to form a composite Z-Score or value metric. Table 6 shows the simulated historical performance, turnover, diversification, capacity and relative factor exposure of the indexes.

Table 6. Performance of Composite Value Indexes: FTSE Developed

	FTSE Developed	EY/SP	EY/SP/CY	EY/SP/CY/BP	EY/SP/CY/BP/DY
Geometric Mean (%)	1.89	3.75	3.68	3.63	3.65
Volatility (%)	17.06	17.90	17.96	18.16	18.05
Volatility Reduction (%)		-4.97	-5.31	-6.45	-5.84
Sharpe Ratio	0.11	0.21	0.20	0.20	0.20
DD (%)	-58.95	-61.72	-61.86	-62.92	-63.16
Two Way Turnover (%)		107.10	111.48	119.70	117.50
Excess (%)		1.83	1.76	1.71	1.73
Tracking Error (%)		2.53	2.72	3.05	3.36
Information Ratio		0.72	0.65	0.56	0.51
Alpha (%)		1.83	1.76	1.72	1.75
Alpha T-Stat		2.81	2.51	2.20	2.00
Beta		1.04	1.04	1.05	1.04
Number of Stocks	1941	1941	1941	1941	1941
Effective N	340	299	309	328	312
Capacity (WCR)	1.00	1.33	1.35	1.41	1.40
Rel. Factor Exposure		0.49	0.51	0.56	0.57

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

As the number of factors decreases, performance metrics improve. This is the result of creating composite measures containing increasingly less relevant factors. In particular the information ratio rises monotonically as the number of factors decrease and is roughly 50% higher for the two-factor composite than for the five factor composite index. Note also, that turnover falls, but not quite monotonically, as the number of factors is reduced.

We base the choice of factors on which to form a composite measure of value on the following considerations:

- Book to Price and Dividend Yield historically have a detrimental effect on the performance and turnover of composite value measures.

- Earnings Yield exhibits a relatively strong correlation with Dividend Yield, both in terms of factor returns and factor characteristics.

- Cash Flow Yield is relatively highly correlated with Book to Price in terms of factor returns and factor characteristics.

This suggests that Dividend Yield and Book to Price may be ignored, since Earnings Yield and Cash Flow Yield are relatively good proxies for them. Consequently, we are left with a choice between a two or three-factor composite measure of value:

- In performance, turnover and capacity terms the two-factor composite measure is slightly superior to the three-factor composite.

- In diversification and factor exposure terms the three-factor composite measure is slightly superior to the two-factor composite.

Given this analysis is very much based "in sample" and the similarity of two and three-factor composite indexes in terms of index characteristics, we have a minor preference for a three-factor construction. This is a cautious choice which ensures greater factor diversity. This choice has the further advantage that our composite factor has a greater orientation towards factors that are less susceptible to manipulation (Cash Flow Yield and Sales to Price) than to those that are (Earnings Yield).

The results to date incorporate monthly index rebalances. From a practical perspective a lower rebalance frequency is required. We illustrate that the simulated historical performance characteristics of our composite three-factor value index is independent of the rebalance frequency and rebalance cycle. Table 7 reports results for a quarterly rebalance, a semi-annual March and September rebalance, a semi-annual June and December rebalance and an annual September rebalance, where the three-factor value tilt is applied to the capitalisation weighted FTSE Developed index.

Table 7. Performance of Composite Value Index: Rebalance Frequency and Cycle

	FTSE Developed	Monthly	Quarterly	Semi-Ann Mar & Sep	Semi-Ann Jun & Dec	Annual
Geometric Mean (%)	1.89	3.68	3.46	3.73	3.52	3.82
Volatility (%)	17.06	17.96	17.87	17.78	17.65	17.60
Volatility Reduction (%)		-5.31	-4.80	-4.22	-3.46	-3.17
Sharpe Ratio	0.11	0.20	0.19	0.21	0.20	0.22
DD (%)	-58.95	-61.86	-62.03	-61.70	-61.37	-61.47
Two Way Turnover (%)		111.48	62.79	44.60	42.46	29.37
Excess (%)		1.76	1.55	1.81	1.60	1.90

|	FTSE Developed	Monthly	Quarterly	Semi-Ann Mar & Sep	Semi-Ann Jun & Dec	Annual
Tracking Error (%)		2.72	2.66	2.78	2.54	2.82
Information Ratio		0.65	0.58	0.65	0.63	0.67
Alpha (%)		1.76	1.56	1.82	1.61	1.91
Alpha T-Stat		2.51	2.25	2.50	2.41	2.56
Beta		1.04	1.04	1.03	1.02	1.02
Number of Stocks	1941	1941	1944	1950	1938	1958
Effective N	341	309	311	311	310	309
Capacity (WCR)	1.00	1.35	1.35	1.35	1.35	1.36
Rel. Factor Exposure		0.51	0.51	0.51	0.51	0.52

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

All index characteristics, including capacity, diversification levels and the degree of factor exposure are robust to the frequency and timing of the rebalance. The rebalance frequency has a substantial impact on turnover. An annual rebalance minimises turnover; we make the pragmatic decision to rebalance annually in September order to synchronise index rebalances with reviews of the underlying index universe and other factor (Quality) indexes.

For comparison, an annual September review applied to the same composite value factor, but from an equally weighted version of the FTSE Developed results in a mean excess return of 1.62%p.a., two-way turnover of 51.74% p.a., an average effective N of 1459, an average WCR of 10.23 and average factor exposure of 0.58.

5. Narrowing of broad value indexes

The composite value indexes examined so far are broad, retaining all underlying index constituent stocks. The aggregate index level factor exposure may be improved, by removing from the broad index, stocks with the smallest contribution to index level factor exposure. Specifically, we calculate the factor contribution of each stock in the broad index (broad value index weight * Z-Score). We then sequentially remove stocks with the smallest factor contribution and recalculate the Effective N, capacity and exposure of the resulting index. This is repeated until the narrow index diversification target (67% of the broad index); or the narrow index capacity target (WCR 2.5x the broad index); or the factor exposure objective (2x the exposure of the broad index relative to the underlying) is achieved.

As the diversification target is reduced, the resulting narrow index becomes increasingly concentrated and a greater exposure to the value factor is achieved. At around two thirds of the broad index Effective N, we achieve noticeable improvements in factor exposure without compromising the diversification levels exhibited by the index.

Table 8 details the result of applying this process to the annually rebalanced broad composite value index derived from the capitalisation weighted FTSE Developed Index.

Table 8. Performance of a Narrow Composite Value Index

	FTSE Developed	Broad	Narrow
Geometric Mean (%)	1.89	3.82	5.41
Volatility (%)	17.06	17.60	18.28
Volatility Reduction (%)		-3.17	-7.17
Sharpe Ratio	0.11	0.22	0.30
DD (%)	-58.95	-61.47	-63.23
Two Way Turnover (%)		29.37	47.86
Excess (%)		1.90	3.45
Tracking Error (%)		2.82	4.62
Information Ratio		0.67	0.75
Alpha (%)		1.91	3.49
Alpha T-Stat		2.56	2.85
Beta		1.02	1.04
Number of Stocks	1958	1958	1396
Effective N	343	309	214
Capacity (WCR)	1.00	1.36	2.42
Rel. Factor Exposure		0.52	0.92

Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

The resulting number of stocks in the narrow index is on average approximately 71% of the FTSE Developed index, but the diversification and capacity constraints are satisfied. The narrow index displays substantially greater levels of exposure to the composite value factor, resulting in an increase in the factor premium, as measured by excess return. However, there is a corresponding increase in turnover. A comparison of Tables 7 and 8 indicates that a narrow composite value index rebalanced annually exhibits almost double the levels of factor exposure (and value premium) compared to broad indexes that are rebalanced more frequently. Turnover levels of the narrow, annually rebalanced index are comparable or lower to broad indexes rebalanced more frequently.

Chart 4 shows the composite value factor exposure through time for the FTSE Developed, annually rebalanced broad and narrow composite value indexes and a monthly rebalanced narrow index. The monthly rebalance provides an indication of the upper limit to value exposure that is achievable. Chart 4 indicates that there is little justification for more frequent rebalances in order to maintain the value exposure of the index. We therefore conclude that an annually rebalanced narrow index is appropriate.

To ensure the results are robust to the timing of the annual rebalance, we examine the outcomes of annual rebalances that follow different rebalance cycles (January to January, February to February, etc). Chart 5 confirms that simulated risk adjusted performance outcomes i.e. Sharpe Ratios and Two-Way Turnover outcomes are insensitive to the timing of the annual rebalance.

Chart 4. FTSE Developed Value Exposure through Time
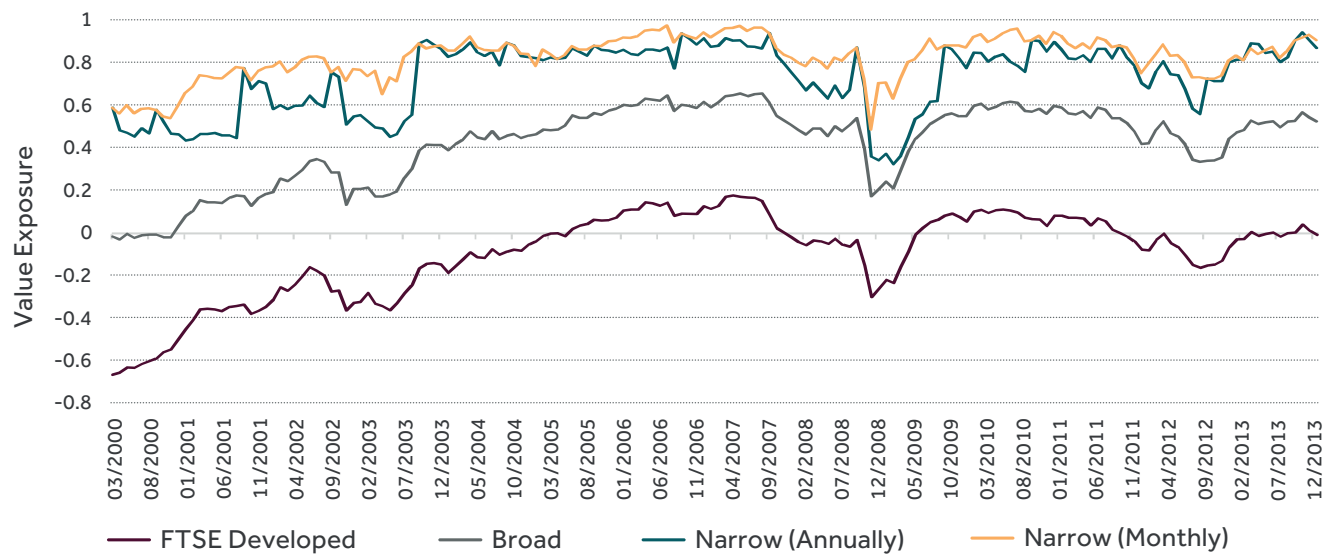
Source FTSE: FTSE Developed capitalisation weighted underlying index; March 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

Chart 5. FTSE Developed; Annual Rebalance Cycle, Sharpe Ratio and Two-Way Turnover
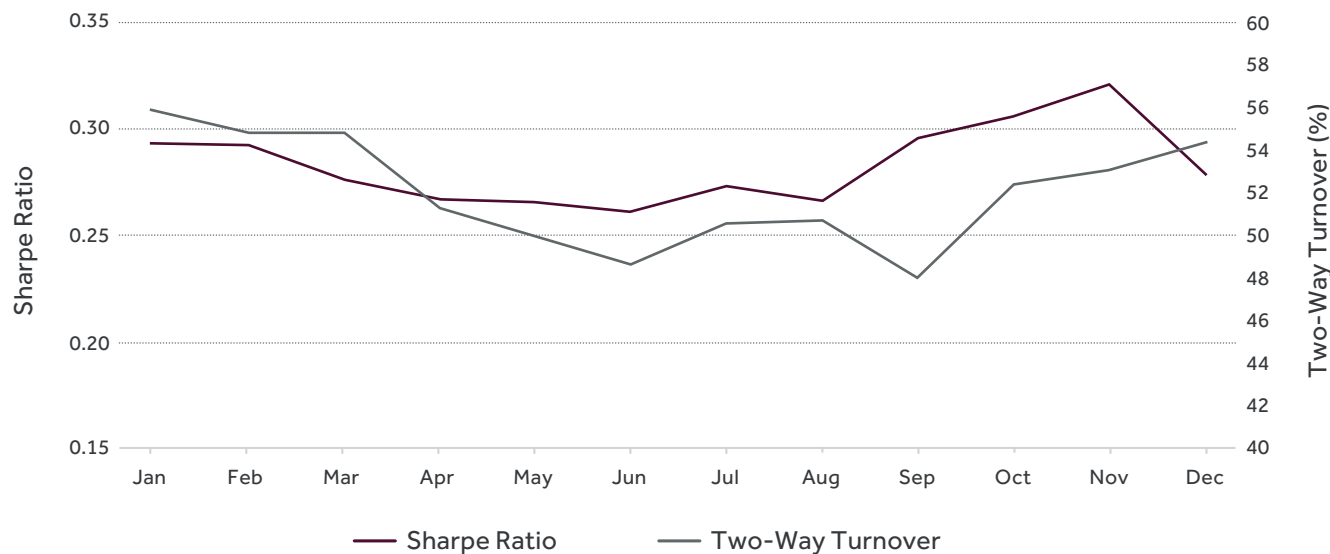
Source FTSE: FTSE Developed; capitalisation weighted underlying index; annual rebalance in different months; May 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

Chart 6 shows the simulated historical performance through time for the FTSE Developed and broad and narrow composite value indexes.

Chart 6. FTSE Developed: Cumulative Performance, Broad and Narrow Value Indexes
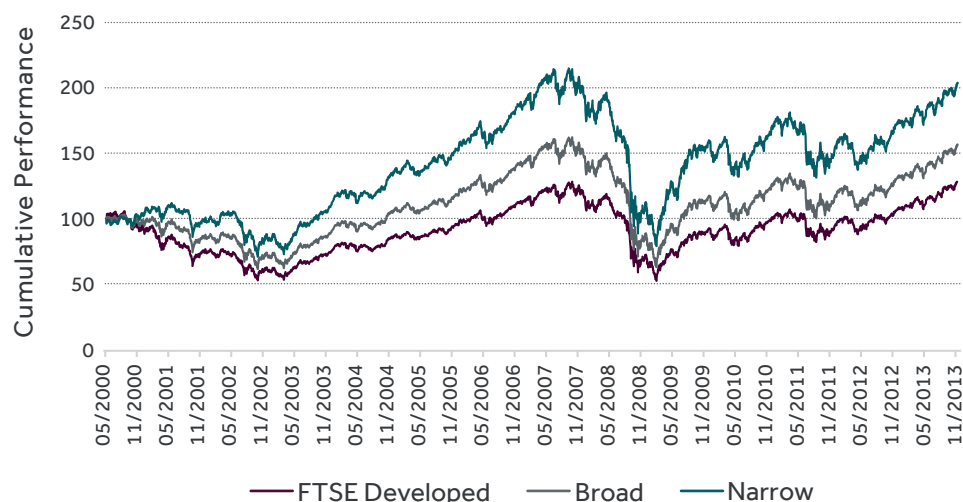
Source FTSE: FTSE Developed; capitalisation weighted underlying index; USD price returns; May 2000 to December 2013. Past performance is no guarantee of future results. Returns shown may reflect hypothetical historical performance. Please see the disclaimer page for important legal disclosures.

6. Conclusions

We examine five common measures of value (Earnings Yield, Book to Price, Cash Flow Yield Sales to Price and Dividend Yield); rejecting two and using three to create a simple composite value factor. The composite measure consists of an equally weighted combination of absolute measures of Earnings Yield and Cash Flow Yield and a country relative measure of Sales to Price.

Over the period May 2000 to December 2013 broad value indexes were constructed by applying a composite value tilt to both a capitalisation weighted and equally weighted versions of the FTSE Developed index.

A broad value factor index rebalanced annually in September results in:

- A value premium of approximately 1.9% p.a. (1.6% p.a. – equal weight).

- Significant and persistent value exposure relative to the underlying index.

- Comparable levels of diversification to the underlying index.

- Index capacity of 74% (67% – equal weight) of the underlying index.

- Approximate annual two-way turnover of 29% p.a. (52% p.a. – equal weight).

Subsequently, we form a narrow value index and achieve increased levels of exposure to the composite value factor:

- A value premium of approximately 3.5% p.a. (capitalisation weighted).

- 77% improvement in factor exposure compared to the broad index.

- Diversification levels of 69% of the broad index.

- Index capacity of 56% of the broad index.

- Annual two-way turnover of 48% p.a.

Appendix A. Regression Analysis

Cross-sectional Regression

Consider the cross-sectional regression:

$$R_i = \alpha + \sum_{j=1}^{J} \delta_{i \subset j} \beta_j + \sum_{k=1}^{K} e_{ki} f_k + \varepsilon$$

where R_i is the return of the ith of N stocks, e_{ki} is the exposure of the ith stock to k^{th} of K factors, $\delta_{i \subset j}$ is industry (country) exposure of the ith stock to j^{th} of J industries (countries), with

$$\delta_{i \subset j} = \begin{cases} 1 & if \quad i \subset j \\ 0 & if \quad i \not\subset j \end{cases}$$

and α is an intercept, β_j is the factor return of jth industry (country), f_k is the factor return of kth factor and ε is the residual. For each stock the industry exposures meet the condition: $\sum_{j=1}^{J} \delta_{i \subset j} = 1$ which acts as a constraint on our regression. This is incorporated in the following way.

Let $\hat{R}_i$ be the return predicted by the model and W_i be some weighting scheme for stocks with $\sum_{i=1}^{N} w_i = 1$. Then employing a weighted regression through minimization of $\sum_{i=1}^{N} w_i (R_i - \hat{R}_i)^2$ implies that

$$\sum_{i=1}^{N} w_i R_i = \sum_{i=1}^{N} w_i \hat{R}_i$$

Further we can exploit freedom to set $\alpha = \sum_{i=1}^{N} w_i R_i$. Taking the weighted sum of both sides of our regression and simplifying now yields:

$$\sum_{i=1}^{N} w_i \sum_{j=1}^{J} \delta_{i \subset j} \beta_j + \sum_{i=1}^{N} w_i \sum_{k=1}^{K} e_{ki} f_k = 0$$

Reversing sums gives

$$\sum_{j=1}^{J} \hat{w}_j \beta_j + \sum_{k=1}^{K} f_k \sum_{i=1}^{N} w_i e_{ki} = 0$$

where $\hat{w}_j$ is the weight in the jth sector. This last equation is solved by setting:

$$\sum_{i=1}^{N} w_i e_{ki} = 0 \text{ and } \sum_{j=1}^{J} \hat{w}_j \beta_j = 0$$

The first of these equations can be satisfied redefining the factor exposure by normalisation thus:

$\hat{e}_{ki} = (e_{ki} - \mu_k) / \sigma_k$ where $\mu_k = \sum_{i=1}^{N} w_i e_{ki}$ and $\sigma_k^2 = \sum_{i=1}^{N} w_i (e_{ki} - \mu_k)^2$. Each normalised factor exposure has mean zero and standard deviation of one.

The second equation can be satisfied by writing the return to industry (country) one as:

$$\beta_1 = -\frac{1}{\hat{w}_1} \sum_{j=2}^{J} \hat{w}_j \beta_j$$

or equivalently by defining a new exposure to industry operator:

$$
\hat{\delta}_{i \subset j} =
\begin{cases}
-\hat{w}_j / \hat{w}_1 & \text{if } i \subset 1 \text{ and } j \neq 1 \\
0 & \text{if } i \subset 1 \text{ and } j = 1 \\
\delta_{i \subset j} & \text{if } i \not\subset 1
\end{cases}
$$

Factor and sector returns are calculated from actual returns R_i, new exposure function to industries (countries), $\hat{\delta}_{i \subset j}$ and normalised exposures to factors $\hat{e}_{ki}$, in the usual way.

Ideal Factor Indexes and Factor Loadings

We define the returns to an "ideal factor index" X as the set of monthly factor returns f_t^x obtained through the cross-sectional regression when we ignore industries (countries), retain the market intercept and choose our value factor as a single factor.

For a given index with monthly excess returns R_t, the factor loading F_i of that index on the ith factor, is given by time series regression:

$$
R_t = \alpha + \sum_i F_i * f_t^{i} + \varepsilon
$$

where α is the intercept and ε is the residual.

Appendix B. Capacity, Diversification and Factor Exposure

Capacity

Let $\hat{w}_i$ be the weights of the index for which we are computing capacity and W_i the weights of the underlying Market Capitalisation weighted index. Then the weighted capacity ratio (WCR) is given by:

$$WCR = \sum_{i=1}^{N} \hat{w}_i * \frac{\hat{w}_i}{w_i}$$

This ratio is bounded below by one and the larger the value, the poorer the capacity.

Let M_i be the market capitalisation of ith stock. Then the weighted average market capitalisation ratio (WAMCR) is given by:

$$WAMCR = \frac{\sum_{i=1}^{N} \hat{W}_i * M_i}{\sum_{i=1}^{M} W_i * M_i}$$

This ratio is bounded below by zero and the larger its value, the better the capacity.

Both definitions maybe used to assess the capacity of an index although we prefer WCR as it does not suffer from the deficiency of implying that an index consisting of one stock with market capitalisation M has the same capacity as an index of N stocks each with market capitalisation M.

Diversification

The diversification measure used in this document is:

$$\text{Effective N} = \left. 1 \middle/ \sum_{i=1}^{M} \hat{W}_i^2 \right.$$

where Effective N is the effective number of stocks in the index, M the actual number of stocks and $\hat{w}_i$ the weight of ith stock in the index.

Factor Exposure

The Factor Exposure of an index is defined to be:

$$\text{Factor Exposure} = \sum_{i=1}^{N} \hat{W}_i * Z_i$$

where $\hat{w}_i$ are the weights of the index and Z_i is the cross-sectional factor Z-Score.

References

1. Asness, C.S., Moskowitz T.J., and Pedersen L.H. "Value and Momentum Everywhere", *Journal of Finance*, v. 68, June 2013, pp. 929 – 985.

2. Basu, S., "Investment Performance of Common Stocks in Relation to their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis", *Journal of Finance*, v. 31, June 1977, pp.663-682.

3. Daniel, K. and Titman, S., "Evidence on the Characteristics of Cross Sectional Variation in Stock Returns", *Journal of Finance*, v. 52, March 1997, pp. 1–33.

4. Fama, E. F., and French, K.R. "The Cross-Section of Expected Stock Returns", *Journal of Finance*, v. 47, June 1992, pp.427-465.

5. Fama, E. F., and French, K.R. "Common Risk Factors in the Returns on Bonds and Stocks", *Journal of Financial Economics*, v. 33, February 1993, pp.3-53.

6. Fama, E. F., and French, K.R. "Size and Book-to-Market Factor in Earnings and Returns", *Journal of Finance*, v. 50, 1995, pp.131-155.

7. "Factor Exposure Indexes – Index Construction Methodology", FTSE (2014)

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