Financial Analysts Journal
Volume 66 • Number 3

Adaptive Asset Allocation Policies

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This article proposes an asset allocation policy that adapts to market movements by taking into account changes in the outstanding market values of major asset classes. Such a policy considers important information, reduces or avoids contrarian behavior, and can be followed by a majority of investors.

The third edition of *Managing Investment Portfolios: A Dynamic Process* states:

> Strategic asset allocation can be viewed as a process with certain well-defined steps. Performing those steps produces a set of portfolio weights for asset classes; we call this set of weights the strategic asset allocation (or the *policy portfolio*).[1]

This article is about such asset allocation policies.

Traditional Asset Allocation Policies

The March 2009 asset allocation report of the California Public Employees' Retirement System (CalPERS)[2] provides the example of a traditional asset allocation policy shown in **Table 1**. A key feature of such a policy is that the target for each asset class is stated as a *percentage of the total value* of the fund, with each asset target between 0 percent and 100 percent. An asset allocation policy is almost always stated in this manner. I use the term "traditional" for such a policy to differentiate it from the adaptive policies described later in the article.

Table 1. CalPERS' Asset Allocation Policy, March 2009

Asset Class	Policy Target
Global equity	66%
Global fixed income	19
Inflation-linked assets	5
Real estate	10
Cash	0

Source: CalPERS (2009).

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A typical large institutional investor sets an asset allocation policy after considerable analysis, changing it only episodically. According to the CalPERS (2009) report:

> CalPERS follows a strategic asset allocation policy that identifies the percentage of funds to be invested in each asset class. Policy targets are typically implemented over a period of several years.

To accommodate disparities between policy proportions and actual portfolio holdings, most traditional asset allocation policies include acceptable ranges around each target weight within which the magnitude of the particular asset class is allowed to vary. For some investors, the deviations can become substantial. At the end of March 2009, the proportions held by CalPERS differed substantially from its policy target (adopted in the latter part of 2007 but still in effect at the time), as shown in **Table 2**.[3]

Table 2. CalPERS' Target and Actual Asset Allocations, March 2009

Asset Class	Policy Target	Current	Current – Target
Global equity	66%	53.5%	–12.5 pps
Global fixed income	19	25.2	+6.2
Inflation-linked assets	5	2.5	–2.5
Real estate	10	11.4	+1.4
Cash	0	7.3	+7.3

pps = percentage points.
Source: CalPERS (2009).

To restore the portfolio by conforming it with the asset allocation policy, CalPERS would have had to sell some of its holdings in three asset classes (global fixed income, real estate, and cash) and purchase additional amounts of two others (global equity and inflation-linked assets). This action was not taken immediately, however, because the CalPERS board was considering a new asset allocation policy at the time.

Although statistics are lacking, most large pension funds, endowments, and foundations appear to have traditional asset allocation policies. In many cases, considerable discrepancies between policy and actual asset proportions are allowed to develop. Some funds actively rebalance holdings to avoid substantial discrepancies, whereas others allow the proportions to change with market movements and then revisit their asset allocation policies when the differences between actual and policy weights become large. Relatively few institutional investors seem to engage in what some might term "slavish" adherence to a set of policy asset weights by engaging in frequent rebalancing transactions.

The majority of multi-asset mutual funds also have traditional asset allocation policies. Unlike many institutional investors, however, many such mutual funds allow only relatively small deviations of the actual asset proportions from those specified in the policy.

Many individual investors invest some or all of their retirement savings in multi-asset mutual funds, either directly or through a 401(k) or other type of retirement plan. Under the Pension Protection Act of 2006, the U.S. Department of Labor[4] includes only two types of mutual or collective funds as "qualified default investment alternatives" (QDIAs): *balanced* (sometimes called *life-stage*) and *target-date* (sometimes called *life-cycle*) funds.[5] At the end of December 2008, 9.1 percent of the $1.084 trillion invested in mutual funds offered by the top 25 providers of such funds to 401(k) plans was invested in balanced or life-stage funds and 8.9 percent was invested in target-date funds.[6]

To illustrate, I provide an example of each type of fund.

Vanguard Balanced Index Fund

With $7.5 billion under management in April 2009, the Vanguard Balanced Index Fund

> seeks—with 60 percent of its assets—to track the investment performance of a benchmark index that measures the investment return of the overall U.S. stock market. With 40 percent of its assets, the fund seeks to track the investment performance of a broad, market-weighted bond index.[7]

The Vanguard Balanced Index Fund compares its returns with those of a benchmark, with 60 percent invested in the MSCI US Broad Market Index and 40 percent in Barclays Capital U.S. Aggregate Bond Index. Over the 36 months ended March 2009, the R^2 value for a comparison of the fund's returns with those of the benchmark was 1.00 (rounded to two decimal places), indicating close conformance of the asset proportions with the 60/40 policy.[8]

Fidelity Freedom 2020 Fund

Fidelity Investments offers a series of target-date funds. Of those funds at the end of December 2008, the Fidelity Freedom 2020 Fund was the one most used by defined-contribution plans, with assets of more than $12 billion from such plans.[9] The next six funds in order of total assets from defined-contribution plans were Fidelity Freedom funds with other target dates.

An excerpt from the 2008 prospectus for the Fidelity family of funds is instructive:

> The following chart [**Figure 1**] illustrates each Freedom Fund's approximate asset allocation among equity, fixed-income, and short-term funds as of March 31, 2008. The chart also illustrates how these allocations may change over time. The Freedom Funds' target asset allocations may differ from this illustration. . . . [Moreover, the fund's adviser] intends to manage each Freedom Fund according to its target asset allocation strategy, and does not intend to trade actively among underlying Fidelity funds or intend to attempt to capture short-term market opportunities. However, [the fund's adviser] . . . may modify the target asset allocation strategy for any Freedom Fund and modify the selection of underlying Fidelity funds for any Freedom Fund from time to time.[10]

A comparison of the allocation for the Fidelity Freedom 2020 Fund at the end of March 2008[11] with that at the end of March 2009[12] is shown in **Table 3**. As intended, over the course of the year, the percentage of value invested in equity had fallen, providing overall asset allocations extremely close to those called for by the "glide paths" shown in Figure 1.

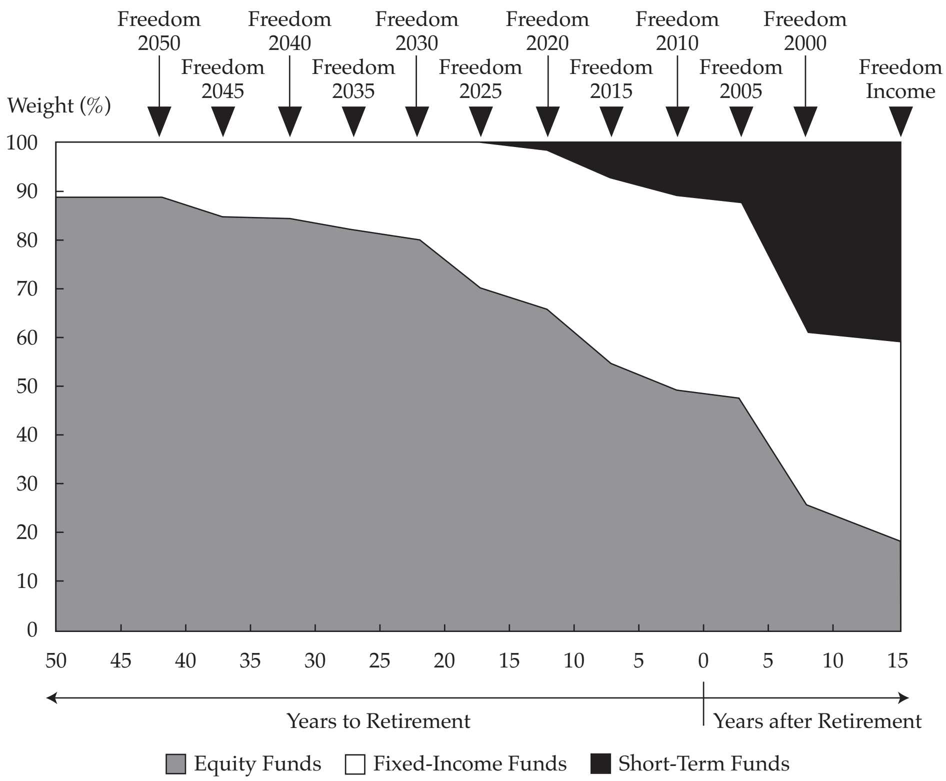
Although these funds provide only examples, their activities suggest that many active and passive multi-asset mutual funds choose to rebalance their holdings significantly after major market movements in order to minimize differences

Table 3. Fidelity Freedom 2020 Fund's Asset Allocations, 2008 and 2009

Asset	Actual, 31 March 2008	Actual, 31 March 2009
U.S. equity	52.6%	52.1%
Non-U.S. equity	13.7	12.9
Investment-grade fixed income	25.5	26.1
High-yield fixed income	7.6	7.6
Short-term funds	0.6	1.3

Source: Fidelity Investments (2008, 2009).

Figure 1. Fidelity Freedom Funds' Asset Allocations



Note: On the *x*-axis, "0" refers to retirement.

Source: Fidelity Investments (2008).

between actual and policy asset allocations. Funds that do so follow traditional asset allocation policies: Balanced funds rebalance to conform with a constant asset allocation over time, and target-date funds rebalance to conform with an asset allocation that varies slowly over time as called for by a prespecified glide path.

The Contrarian Nature of Traditional Asset Allocation Strategies

The term "contrarian" is used in many contexts. The following definition is closest to the meaning I intend in this article:

> An investment style that goes against prevailing market trends by buying assets that are performing poorly and then selling when they perform well.[13]

For purposes of this article, I consider investors contrarian if they buy assets that perform poorly relative to the other assets in the portfolio and sell assets that perform well relative to the others.

Consider an investor who attempts to keep the actual asset percentages of a portfolio consistent with a stated asset allocation policy. I define such a strategy as one that *follows an asset allocation policy* by rebalancing a portfolio frequently to conform it with a pre-specified set of asset proportional values. Assume that our investor rebalances a portfolio to conform it with a stated set of asset proportions at the end of every review period (e.g., every month or quarter). Given *n* asset classes, the dollar amounts initially invested in the assets are $X_1, \ldots, X_n$. The initial value of the portfolio is

$$V_0 = \sum_i X_i, \tag{1}$$

and the initial asset proportions are

$$\frac{X_1}{V_0}, \ldots, \frac{X_n}{V_0}. \tag{2}$$

We assume that these proportions are equal to the investor's asset allocation policy proportions.

Now imagine that a period has passed and that the *value relative* for asset i (the ratio of the ending value to the beginning value) is k_i. The new dollar values of the assets are

$$k_1 X_1, \ldots, k_n X_n. \tag{3}$$

The ending value of the portfolio is

$$V_1 = \sum_i k_i X_i, \tag{4}$$

and the new asset proportions are

$$\frac{k_1 X_1}{V_1}, \ldots, \frac{k_n X_n}{V_1}. \tag{5}$$

We denote the value relative for portfolio K_p as

$$K_P \equiv \frac{V_1}{V_0}. \tag{6}$$

Now assume that the investor wishes to purchase and sell securities in amounts that will make the new asset proportions equal the initial policy proportions. Let $D_1, \ldots, D_n$ represent the dollar amounts of the assets purchased (if positive) or sold (if negative). The goal is to select a set of positive, negative, and possibly zero values for $D_1, \ldots, D_n$ such that

$$\frac{k_i X_i + D_i}{V_1} = \frac{X_i}{V_0} \tag{7}$$

for every asset i. This step requires that

$$D_i = \left(K_p - k_i\right) X_i. \tag{8}$$

Also of interest is the amount of an asset purchased as a proportion of the value before the transaction. We can denote this amount as Y_i:

$$Y_i \equiv \frac{D_i}{k_i X_i} = \frac{K_p}{k_i} - 1. \tag{9}$$

If an asset underperforms the portfolio as a whole, $(K_p - k_i)$ will be positive. As Equation 8 shows, the investor will purchase the asset because D_i will be positive. Such assets are *relative losers*. Moreover, the poorer such an asset's performance (i.e., the smaller its value relative, k_i), the greater will be Y_i, the amount purchased, as a percentage of the current holding.

Conversely, if an asset outperforms the portfolio as a whole, $(K_p - k_i)$ will be negative. The investor will sell the asset because D_i will be negative. Such assets are *relative winners*. Moreover, the better such an asset's performance (i.e., the larger its value relative, k_i), the greater will be Y_i, the amount sold, as a percentage of the current holding.

In this setting, an investor who follows an asset allocation policy is undoubtedly a contrarian. To repeat the obvious:

> Rebalancing a portfolio to a previously set asset allocation policy involves selling relative winners and buying relative losers.[14]

Contrarians All?

If I wish to buy a security, someone must sell it to me. If I wish to sell a security, someone must buy it. Anyone who rebalances a portfolio to conform with an asset allocation policy must trade with someone.

From time to time, companies and other entities issue new securities and purchase or redeem existing ones. But most security transactions involve trades of existing securities between two investors, which raises the question, can all investors be contrarians? The answer is no.

I illustrate with a simple example. Each of four investors follows an asset allocation policy with positive proportions of four asset classes, although the proportions differ. A period has passed, and the assets have performed differently. In **Table 4**, the assets are numbered in terms of their performance (i.e., $k_1 > k_2 > k_3 > k_4$). The investors differ in their initial allocations and thus have different overall portfolio returns (K_p values). Each investor wishes to make transactions to rebalance the portfolio to the particular asset allocation policy. In Table 4, a minus sign indicates an asset to be sold and a plus sign one to be purchased. The last three columns show the number of investors wishing to sell an asset, the number wishing to buy, and the difference between the two.

Because every investor holds the best performing asset, every portfolio return will be below the return of the best performing asset. Hence, every investor will wish to sell shares of Asset 1. Conversely, every portfolio return will be greater than the return of the worst performing asset, and

Table 4. Asset Allocation Trades for Four Investors

Assets in Decreasing Order of Return	Investor A	Investor B	Investor C	Investor D	No. of Sellers	No. of Buyers	Net No. of Sellers
1	–	–	–	–	4	0	4
2	–	–	–	+	3	1	2
3	+	+	–	+	1	3	–2
4	+	+	+	+	0	4	–4

thus every investor will wish to buy shares of Asset 4. With regard to the best and worst performing assets, every investor is indeed a contrarian, and the group as a whole must find some investors who do not follow asset allocation policies with whom to trade.

Note that in each investor's column, minus (sell) signs come first, followed by plus (purchase) signs because each investor will wish to sell all assets with performance (k_i) greater than that of the portfolio (K_p). Investor asset allocations, like portfolio returns, will differ, however, so the points at which minus signs stop and plus signs begin will vary—all of which leads to a key characteristic of the last column: The net number of sellers (number of sellers – number of buyers) will be smaller, the poorer an asset's performance.

To keep the example simple, I assume that each investor's portfolio performance differs from that of each asset. This assumption, however, need not be the case. If an asset's performance equals that of an investor's portfolio, the investor will not wish to buy or sell shares in it—a situation that could be represented with a zero in the table.

We could make another modification that would increase the realism of the example. Some investors may have an asset allocation policy that calls for zero exposure to one or more assets. This situation could also be represented with a zero in the table because no trades will be required.

Taking such possibilities into account would modify the characteristics of the table only slightly. The net number of sellers will either decrease or stay the same, the lower an asset's performance.

We should not read too much into this result. Although the net *number* of sellers will not increase the lower an asset's return, the difference between the *dollar value* of shares offered for sale and the dollar value of shares desired to be purchased by the group of investors that follows asset allocation policies may not share this characteristic because of differences in the values of an asset's holding across portfolios. Put somewhat differently, the relationship between (1) the net number of shares offered and (2) asset return may not be completely monotonic, especially for assets with returns close to that of the overall market.

Despite this caveat, those attempting to rebalance portfolios to asset allocation policies will, as a group, wish to sell shares of the best performing asset and purchase shares of the worst performing asset. This fact alone leads to two conclusions that should seem obvious at this point:

> Not all investors can be contrarians.

Thus:

> Not all investors can follow traditional asset allocation policies.[15]

More pragmatically, for a large number of investors to be able to follow traditional asset allocation policies, a large number of other investors must be willing to take the other sides of the requisite trades. Investors in the latter group must purchase assets that have performed well (relative winners) and sell assets that have performed poorly (relative losers). As I discuss later in the article, such a strategy will prove superior if security price trends persist; therefore, investors who pursue such a strategy are often termed *trend followers*. To oversimplify, for every contrarian there must be a trend follower. Not only is it impossible for all investors to follow contrarian strategies, but it is also impossible for those with a majority of capital assets to do so.

Identifying investors who have traditional asset allocation policies is easy. As indicated earlier, there are many such investors. But identifying investors who pursue trend-following policies is harder. This fact raises a more practical question: How many investors actually follow an asset allocation policy? The answer might well be relatively few. Although many investors have asset allocation policies, relatively few are likely to follow their policies by rebalancing their portfolios frequently. As suggested earlier, multi-asset mutual funds appear to be a major exception: They rebalance their portfolios frequently by buying relative losers and selling relative winners.

Why a Contrarian Strategy?

Why might an investor wish to adopt a contrarian strategy? There are two possible reasons. The investor might believe that markets are *efficient* and that the preferences and/or positions of the ultimate beneficiary or beneficiaries of a fund differ sufficiently from those of the average investor to warrant such a strategy. Alternatively, the investor might believe that markets are *inefficient* and that the majority of investors do not realize that a contrarian strategy can provide a better combination of risk and return than can conventional trend-following strategies.

Efficient-Market Views. Perold and Sharpe (1988) documented a key relationship between market returns and the performance of different asset allocation policies. They compared the payoffs provided by following a traditional asset allocation policy with those obtained by following a buy-and-hold strategy. Assuming investments in two asset classes (bills and stocks), they defined *constant-mix strategies* as those that "maintain an exposure to stocks that is a constant proportion of wealth" (p. 18). They noted:

> In general, rebalancing to a constant mix requires the purchase of stocks as they fall in value . . . and the sale of stocks as they rise in value . . . where, strictly speaking, changes in value are measured in relative terms. (pp. 19–20)

Perold and Sharpe (1988) further showed that the desirability of rebalancing to constant proportions of wealth depends on the movements of market prices:

> In general, a strategy that buys stocks as they fall and sells as they rise will capitalize on reversals. The marginal purchase decisions will turn out to be good ones, as will the marginal sell decisions. A constant-mix strategy will thus outperform a comparable buy-and-hold strategy in a flat (but oscillating) market precisely because it trades in a way that exploits reversals. . . . [But] a constant-mix approach will underperform a comparable buy-and-hold strategy when there are no reversals. This will also be the case in strong bull or bear markets when reversals are small and relatively infrequent, because most of the marginal purchase and sell decisions will turn out to have been poorly timed. . . .
>
> Cases in which the market ends up near its starting point are likely to favor constant-mix strategies, while those in which the market ends up far from its starting point are likely to favor buy-and-hold strategies. . . . Neither strategy dominates the other. A constant-mix policy tends to be superior if markets are characterized more by reversals than trends. A buy-and-hold policy tends to be superior if there is a major move in one direction. (pp. 21–22)
>
> Ultimately, the issue concerns the preferences of the various parties that will bear the risk and/or enjoy the reward from investment. There is no reason to believe that any particular type of dynamic strategy is best for everyone (and, in fact, only buy-and-hold strategies could be followed by everyone). (p. 26)

Roughly speaking, an efficient-market view holds that an investor is best served by adopting the average opinion of investors about the probabilities of possible future combinations of returns. Among investors who accept this premise, the return distribution associated with a rebalancing strategy will appeal to only a minority, with another group of investors taking the other sides of the rebalancing trades of the first group. Absent superior knowledge about the return-generating process, investors should follow a traditional asset allocation policy only if they are less concerned than the average investor about inferior returns in very bad or very good markets. This scenario seems an unlikely one for the typical small investor, for whom most balanced and target-date funds are designed.

From late 2007 through early 2009, returns on stock markets around the world were dismal, with many markets posting losses of 50 percent or more in real terms. Sobered by these results, some analysts changed their assumptions about stock returns. In some cases, positive serial correlations of returns were assumed. This assumption increased the probabilities of trends and thus extreme long-term returns. In other cases, some other process was included to provide a distribution with a "fat left tail," which increased the probabilities of large negative returns. Some analysts included both features in their models. In models with such assumptions, extreme markets are more likely, making traditional asset allocation policies even less appropriate for funds designed for small investors.

Inefficient-Market Views. Many advocates of rebalancing rationalize their position by assuming that markets are inefficient and that other investors with whom they can trade do not fully understand the nature of asset returns. As Arnott and Lovell (1990) opined:

> How many investors permit their asset mix to drift with the whims of the markets (assuring overweighting at market highs and underweighting at the lows). . . . Simple rebalancing can provide the necessary measure of control over a drifting mix. It is worthwhile if properly managed. (pp. 13, 18)

Note the reference to "market highs . . . and lows." The statement suggests that one can tell when an asset is at its market high or low before the fact—hardly an efficient-market concept.

Although Arnott, Burns, Plaxco, and Moor (2007) took a more nuanced approach, they still seemed to suggest that rebalancing can take advantage of market inefficiency:

> Not rebalancing may mean holding assets that have become overpriced, offering inferior future rewards. A commitment to rebalance to the strategic asset allocation offers an effective way to dissuade clients from abandoning policy at inauspicious moments. (p. 702)

To buttress their view, Arnott et al. (2007) reported the results of an empirical test that used monthly rebalancing from 1973 to 2003, which showed that a rebalanced portfolio would have provided a greater average return with a smaller standard deviation than would a "drifting mix." As discussed earlier, rebalancing to a constant mix typically outperforms a buy-and-hold strategy when reversals are more common than trends. In periods with more trends than reversals, the comparison is likely to yield the opposite conclusion. As is frequently the case, the outcomes of empirical tests with past data can be highly period dependent.

When adopting an investment strategy, one must make an assumption about the nature of future security markets. If one believes that markets are inefficient, taking advantage of investors who do not realize that this is so makes sense. Nonetheless, the task can be daunting, as Arnott (2009) argued:

> At its heart, rebalancing is a simple contrarian strategy. In ebullient times, this means taking money away from our biggest winners. In the worst of times, the process forces us to buy more of the assets that have caused us the greatest pain. Most investors acknowledge it as a critical part of the successful investor's toolkit. But recognition and action are two different things. Surrounded by bad news, pulling the trigger to buy securities down 50 percent, 75 percent, or even 90 percent is exceedingly difficult for even the staunchest of rebalancers. Many lose their nerve and blink, letting a healthy portion of the excess returns slip from their grasp. (p. 1)

Arnott's argument reflects some of the points I have made thus far. It recognizes that rebalancing is, in fact, a contrarian strategy. It acknowledges that such action involves buying losers and selling winners. It suggests that most investors believe rebalancing is desirable but that many "lose their nerve and blink." And it reflects Arnott's view that markets are sufficiently inefficient that by failing to rebalance, investors let "a healthy portion of the excess returns slip from their grasp."

Asset Allocation Policy and Market Efficiency

The vast majority of those who adopt an asset allocation policy heed the following recommendations:

> The expectations involved in strategic asset allocation are long term. "Long term" has different interpretations for different investors, but five years is a reasonable minimum reference point.[16]

Are markets efficient in the long run? That depends on what is meant by the term "efficient." In the current context, we need merely ask whether an investor wishes to assume that significant numbers of investors are foolish enough to take the other side of contrarian trades when doing so is undesirable. An investor who adopts a traditional asset allocation policy and rebalances frequently to conform with it must either (1) have an unusual set of preferences for returns in different markets (as described earlier) or (2) believe that markets will be inefficient in this sense more often than not over a period of several years.

I believe that the majority of institutional investors who adopt traditional asset allocation policies do so for neither of those two reasons. Rather, they adopt a policy designed to reflect both their preferences for risk vis-à-vis return and their special circumstances when they adopt a policy. As time passes and markets change, the policy no longer serves its original purpose. But neither a traditional rebalancing approach nor a "drifting mix" is appropriate. I suggest two possible alternatives later in the article. First, however, let us see how far a traditional policy can diverge from its original position.

Bond and Stock Values in the United States

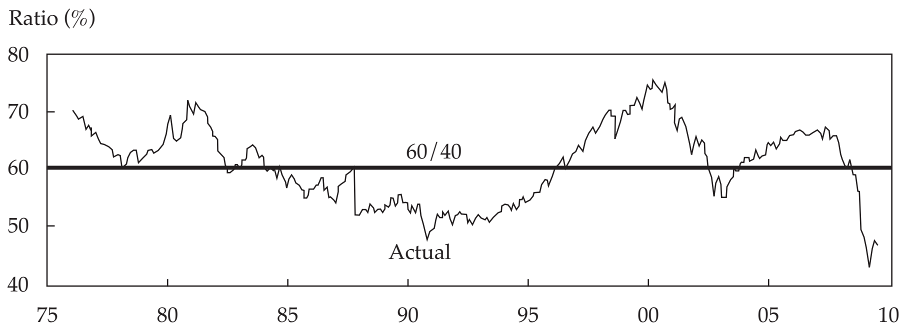
Consider a simple asset allocation policy that involves only U.S. bonds and U.S. stocks. Assume that the former are represented by the Barclays Capital (formerly Lehman) U.S. Aggregate Bond Index and the latter by the Wilshire 5000 Total Market Index.[17] Now, consider a balanced mutual fund that has chosen an asset allocation policy with 60 percent invested in U.S. stocks and 40 percent in U.S. bonds and that uses these two indices as benchmarks. Its goal is to provide its investors with a portfolio representative of the broad U.S. market of stocks and bonds. Investments in each asset class are made via index funds in order to track the underlying returns closely.

The similarity of this fund to the Vanguard Balanced Index Fund is not coincidental. The two differ only with respect to the indices used for U.S. stock returns, but the two alternatives are highly correlated.

Figure 2 shows the ratio of (1) the total market capitalization of the stock index to (2) the sum of the total market capitalizations of the bond and stock indices over the period January 1976–June 2009. More succinctly, it shows the value of U.S. stocks as a percentage of the value of U.S. stocks and bonds over 33.5 years.

As Figure 2 shows, the relative values of U.S. stocks and bonds have varied substantially. This finding is not an exception—in other countries, security values have also varied substantially.

Figure 2. The Ratio of the Value of U.S. Stocks to U.S. Stocks plus Bonds, January 1976–June 2009



Over the entire period, the proportion of value of stocks averaged 60.7 percent—close to that of a traditional 60/40 strategy with monthly rebalancing, as shown in the figure. The average increase in the value of bonds was larger than that of stocks. But the total return on stock investments averaged more than that on bond investments, as one might expect over the long run as a reward for the greater risk of stock investments.

Table 5 shows the annualized monthly averages[18] of the total returns, the percentage changes in market value, and the differences between the two. Overall, investors neither extracted large amounts of cash from the bond and stock markets nor invested substantial amounts of new cash. They did, however, invest in new bonds in amounts that were close to the sum of coupon payments received from bonds and the dividends paid by their stocks.

Table 5. Returns and Changes in Market Value of U.S. Bonds and Stocks, January 1976–June 2009

	Return	Change in Market Value	Difference
Bonds	8.20%	10.82%	2.62 pps
Stocks	11.27	8.96	–2.31

pps = percentage points.

Assume that our balanced fund opened its doors in February 1984, when the value of U.S. stocks was 59.62 percent of the total value of stocks and bonds. At the time, the fund with 60 percent in stocks well represented an investment in the U.S. bond and stock markets and should have had a similar risk and expected return. Fast-forward to October 1990. The market value of stocks is now 47.99 percent of the total, but the fund has been rebalanced to maintain its policy target of 60 percent. It is no longer representative of the market's risk and return; instead, the fund is riskier, presumably with a higher expected return.

Figure 2 shows that from January 1976 through June 2009, our fund varied from being significantly riskier than the U.S. bond plus stock market to being considerably less risky. At the end of March 2000, the proportion of market value in stocks was 75.06 percent, leading to the lowest relative risk for the fund in the entire period. At the end of February 2009, the situation was just the opposite: The proportion of market value in stocks fell to its nadir of 43.18 percent, making the fund, at 60 percent, much riskier than the overall U.S. bond plus stock market.

In sum, our fund failed to meet its goal of providing a strategy representative of the overall U.S. bond and stock markets except in the very long run. And, as Keynes (1923) taught us, "in the long run we are dead" (ch. 3).

To accomplish its goal, our fund needs to *adapt* its allocation policy. Let us now consider two approaches that an investor might use: (1) optimization based on reverse optimization and (2) an approach that I call an adaptive asset allocation policy. We will assume that the investor is concerned with only the return on assets (ruling out cases in which liabilities are taken into account) and that the managed fund constitutes the entire portfolio (ruling out the use of balanced or target-date funds as components of a larger portfolio).

Optimization Based on Reverse Optimization

Many asset allocation policies are chosen after extensive analyses designed to determine a set of optimal strategies with different combinations of risk and return. In some cases, the analysis uses a standard Markowitz mean–variance approach. In others, the goal is to maximize an investor's expected utility. In many cases, these optimization analyses are conducted with constraints on asset holdings that are designed to reflect liquidity requirements or other factors. Moreover, the chosen policy may differ to some extent from the analytically "optimal" asset mixes.

Whatever the process, asset allocation policies are set after considering estimates of the risks and returns of major asset classes and the correlations among their returns. More generally, the relationship can be characterized as follows:

$$\text{Asset allocation}_t = f\left(\text{Investor characteristics}_t, \text{Market forecasts}_t\right). \quad (10)$$

The subscripts indicate that the appropriate asset allocation at time *t* depends on the investor's characteristics and the forecasts for asset returns and risks at the time. The notation $f()$ should be read as "is a function of" the items in parentheses.

Consultants and others who make market forecasts typically consider historical returns and some aspects of economic theory. Some forecasters, but by no means all, consider the current market values of major asset classes. The following rather crude equation represents the preferred approach:

$$\text{Market forecasts}_{>t} = f\left(\text{History}_{<=t}, \text{Economic theory}_t, \text{Market values}_t\right). \quad (11)$$

The subscripts indicate that market forecasts for outcomes occurring after time *t* are based on historical information for periods up to and including time *t* and on economic theory and market values at time *t*.[19]

Why should market values inform forecasts? Because an asset's current market value reflects the collective view of the probabilities of future prospects. This information is valuable and should be taken into account when managing a portfolio.

Analytic approaches for making market forecasts in this manner are generally termed *reverse optimization*.[20] Mean–variance approaches assume that capital markets provide unbiased estimates of future prospects (Sharpe 1974, 1985) or incorporate views about deviations from such estimates (Black and Litterman 1991). A comparable approach has been suggested for making forecasts to be used in expected utility analyses (Sharpe 2007).

Combining Equations 10 and 11 gives the following important relationship:

$$\text{Asset allocation}_t = f\left(\text{Investor characteristics}_t, \text{History}_{<=t}, \text{Economic theory}_t, \text{Market values}_t\right). \quad (12)$$

The inevitable conclusion is that an investor's asset allocation, expressed in the traditional manner as percentages of total value in each asset class, should change over time to reflect changing market values, even if the investor's characteristics are unchanged. This conclusion is the key tenet of this article.

Note that such an approach may not require substantial transactions over and above those associated with reinvesting dividends, interest payments, and other cash received from security issuers. Moreover, there is no reason to expect that such an approach will involve selling relative winners and buying relative losers, as must be done to conform with a traditional asset allocation policy.

Equation 12 also shows that a formal system could be set up to revise an investor's asset allocation policy frequently by conducting a reverse optimization analysis followed by optimization. This process, however, requires complex models in order to accommodate real-world aspects[21] and is apparently used by relatively few organizations.[22]

To provide an alternative, I offer a procedure that can be used to periodically adapt an organization's asset allocation policy in light of asset market values without requiring actions that are clearly contrarian in nature. The procedure is simple and can be implemented easily. Although I make no claim that it is the best possible approach, it should be better than either strict conformance with a traditional asset allocation policy or the adoption of such a policy followed by subsequent actions (or lack thereof) that treat the policy as irrelevant.

Adaptive Asset Allocation Policies

The term "adapt" can be defined as follows:

> To adjust oneself to different conditions, environment, etc.[23]

In this case, the "different conditions, environment, etc." are new asset market values.

Imagine that a fund investing in U.S. stocks and bonds established an asset allocation policy of 80 percent stocks and 20 percent bonds at the end of February 1984, with the Wilshire 5000 Total Market Index representing stocks and the Barclays Capital U.S. Aggregate Bond Index representing bonds (as shown in Figure 2, the market proportions were 59.62 percent and 40.38 percent, respectively). Assume that this information was taken into account in the study that led to the 80/20 policy.

A traditional approach would hold that the policy calls for adjustments in holdings to achieve an 80/20 allocation no matter what the subsequent market proportions might be. But as I have argued, this course is unlikely to be a wise one. Instead, the policy proportions should be adjusted as market values change. I propose instead a procedure that I call an adaptive asset allocation (AAA) policy.

Table 6 and **Table 7** show the required calculations for the appropriate allocation at the end of October 1990, when stocks were a substantially smaller portion of overall market value.

Table 6. Market Values of U.S. Bonds and Stocks, February 1984 and October 1990

	Stocks ($ billions)	Bonds ($ billions)	% Stocks
$V_{im,0}$ (Feb. 1984)	1,648.19	1,116.49	59.62
$V_{im,t}$ (Oct. 1990)	2,652.92	2,875.69	47.99
$k_i = V_{im,t}/V_{im,0}$	1.6096	2.5757	

Table 7. Adaptive Policy Allocations, February 1984 and October 1990

	Stocks	Bonds	Sum
$AA_{if,0}$	80.00%	20.00%	
$(AA_{if,0})(k_i)$	128.77	51.51	180.28%
$(AA_{if,0})(k_i)/\text{Sum}(AA_{if,0})(k_i)$	71.43	28.57	

Table 6 shows the total outstanding market capitalizations for each of the two indices at the two dates (February 1984 [time 0] and October 1990 [time t]) and the ratios of the ending values to the beginning values (denoted k_i, as before). Table 7 shows the calculations for the new asset allocation. For each asset, the initial proportion in the fund is multiplied by the ratio of the new total market value of the asset class as a whole to the old value. In this case, the outstanding values of both assets increased substantially. Hence, the sum of the adjusted proportions for the fund is much greater than 100 percent, as shown in the third column. To compute the new asset value proportions for the fund, the figures in the second row are divided by their sum, which gives the new asset allocation shown in the last row.

In this case, stocks fell from representing close to 60 percent of total market value to slightly less than 48 percent. Thus, the fund's asset allocation policy changed from one with 80 percent invested in stocks to one with 71.43 percent so invested.

Why is this procedure likely to be preferred to a traditional approach? Because it need not require that investors who have such policies transact with other investors whenever market prices change, as do traditional asset allocation policies.

Consider a world in which changes in asset market values result only from changes in security prices and reinvestment of cash flows from each asset in the same class. An investor who makes no withdrawals or additional investments and chooses to reinvest all cash flows from each asset class in the same class will be in compliance with the particular adaptive policy at all times and will not need to transact with other investors. This scenario follows from the fact that the value of the investor's holdings of each class will change by precisely the same percentage as the value of the market (k_i) will change. In such a setting, AAA policies are *macro-consistent* in the sense that all investors can follow such strategies.

Of course, the investment world is extremely complex. New issues of securities, buybacks, and redemptions occur frequently. Moreover, the total values of these transactions for an asset class rarely net to zero. Public companies go private and vice versa. Some investors have positive net cash flows that require purchases of new assets, whereas others must sell assets to raise cash. Despite these complications, a group of investors that follows AAA policies is unlikely to need to make large purchases or sales of assets with investors who do not follow such policies. This situation contrasts starkly with the situation of investors who attempt to comply with traditional asset allocation policies; such investors must frequently purchase assets that are relative losers and sell those that are relative winners.

Tables 6 and 7 use the following formula to compute the proportion invested in asset i in fund (f) at time t:

$$X_{if,t} = \frac{X_{if,0}\left(V_{im,t}/V_{im,0}\right)}{\sum_i X_{if,0}\left(V_{im,t}/V_{im,0}\right)}. \quad (13)$$

Here, $V_{im,t}$ and $V_{im,0}$ are the total outstanding market values of asset i at times t and 0, respectively. The proportion invested in asset i at time 0 is $X_{if,0}$, and the proportion to be invested at time t is $X_{if,t}$.

Note that the total market value of asset i at time 0 equals its proportion of total value ($X_{im,0}$) times the total value of all assets at the time ($V_{m,0}$). A similar relationship holds for time t. Thus,

$$\begin{aligned} V_{im,0} &= X_{im,0}V_{m,0} \\ V_{im,t} &= X_{im,t}V_{m,t}. \end{aligned} \quad (14)$$

Substituting these relationships in Equation 13 and canceling terms gives a formula for calculating the new proportion for the fund to invest in asset i

as a function solely of the proportions at time 0 and of the ratios of the proportions for the market portfolio at the two periods:

$$X_{if,t} = \frac{X_{if,0}\left(X_{im,t}/X_{im,0}\right)}{\sum_i X_{if,0}\left(X_{im,t}/X_{im,0}\right)}. \quad (15)$$

Table 8 shows the computations for February 1984 and October 1990.

Table 8. Adaptive Policy Allocations with Asset Proportions, February 1984 and October 1990

	Stocks	Bonds	Sum
$X_{im,0}$	59.62%	40.38%	
$X_{if,0}$	80.00	20.00	
$X_{im,t}$	47.99	52.01	
$(X_{if,0})(X_{im,t})/X_{im,0}$	64.39	25.76	90.15%
$X_{if,t}$	71.43	28.57	

In this example, the initial asset allocation set at time 0 is assumed to have been appropriate, given the market values of the asset classes at the time. This allocation is typically the case when an institutional investor selects an asset allocation policy. The adaptive formula (Equation 15), however, can be applied in other contexts. Key is the statement of an asset allocation policy in terms of base values for (1) the policy and (2) the market. A possible example would be the following:

> The fund's Asset Allocation Policy is to have 80 percent invested in stocks and the remainder in bonds when the market value of stocks is 60 percent of the total value of stocks and bonds, with the proportions to be determined each period by using the adaptive asset allocation formula.

Table 9 shows this asset allocation policy in terms of the formula (Equation 15).

Table 9. An Asset Allocation Policy

	Stocks	Bonds
$X_{im,0}$	60%	40%
$X_{if,0}$	80	20

More generally, we can denote the mixes of market and fund assets as

$$X_{m,0} = \left(X_{1m,0}, X_{2m,0}, \ldots, X_{nm,0}\right) \quad (16)$$

and

$$X_{f,0} = \left(X_{1f,0}, X_{2f,0}, \ldots, X_{nf,0}\right), \quad (17)$$

where there are n assets and $X_{m,0}$ and $X_{f,0}$ are vectors representing the *base* market and fund asset allocations, respectively.

With this interpretation, we can straightforwardly apply the adaptive approach to a wide range of investments. I illustrate with three prototypical types of funds.

Institutional Funds

As indicated earlier, most pension funds, endowments, and foundations conduct asset allocation studies that lead to the selection of a policy asset mix, which can be considered the base policy ($X_{f,0}$). Presumably, this asset allocation was considered appropriate given the market conditions ($X_{m,0}$) at the time, whether or not these market values were used explicitly when determining asset prospects. Under this assumption, the asset allocation policy can be converted to an adaptive policy by simply applying the adaptive formula (Equation 15) in subsequent periods.

As with the traditional approach, allowable deviations from the policy targets may be selected. For example, these deviations could be set to equal the currently allowed deviations of the holdings in each asset class. If the fund makes few, if any, trades, the resulting ranges are less likely to be violated under an adaptive policy than under a traditional approach.

Undoubtedly, many institutional funds would consider the conversion from a traditional to an adaptive asset allocation policy too dramatic to accept overnight. At the very least, I would suggest that the required computations for an adaptive asset allocation policy be performed periodically so that key decision makers could evaluate the fund's holdings in terms of both the traditional approach and this alternative approach. In time, such an exercise might lead to a greater acceptance of the latter.

Balanced Funds

As previously discussed, multi-asset mutual funds are likely to make transactions that are required by their traditional asset allocation policies. To a considerable extent, a multi-asset mutual fund's asset allocation policy will drive its investments, which makes the choice of the type of policy especially important.

Typically, a balanced fund is designed to provide a mix of two or more asset classes with a constant level of "conservatism" or "aggressiveness." Although some may think of these terms as absolute, a more pragmatic approach is to interpret them as relative. In this view, an "aggressive" fund should provide more risk than the market as a whole, whereas a "conservative" one should provide less. A "representative" fund could be designed to provide the same risk as the market as a whole.

Constructing an adaptive representative fund is easy. It would have the market proportions of the asset classes in every period. At the time of formation, the actual proportions ($X_{f,0}$) would be set to equal the market proportions ($X_{m,0}$). The adaptive formula (Equation 15) would subsequently lead the fund manager to hold assets in market proportions at each period. For example, a balanced fund designed to represent the U.S. stock and bond markets would follow the curve rather than the horizontal line in Figure 2.

An aggressive fund would begin with an asset mix that had greater risk than that of the market at the time and would then adjust its holdings by using Equation 15. **Figure 3** shows the actual proportions for a fund that wishes to hold an 80/20 mix of U.S. stocks and bonds when the market proportions are 60/40.

As intended, the fund remained more aggressive than the market throughout the period. Nevertheless, the ratio of the fund's proportion in stocks to that of the market varied, beginning at 1.333 (80 percent/60 percent) and ranging between 1.18 and 1.55.

Not surprisingly, at the four times when the total market value of stocks was close to 60 percent of the total value of stocks and bonds, the fund's asset allocation policy dictated a value close to the intended 80/20 mix. In this example, although the fund was started when the market mix was 60/40, it could have been started at any time with a policy stated in terms of a "normal mix" ($X_{f,0}$) when markets are "normal" ($X_{m,0}$).

This scenario suggests a simple way to convert an existing balanced fund to an adaptive one. The stated policy ($X_{f,0}$) need only be augmented by the "normal" market conditions ($X_{m,0}$) for which it is appropriate. The asset allocation policy for any period can then be determined by using the adaptive formula (Equation 15).

Target-Date Funds

As discussed earlier, a target-date fund has a policy "glide path" that indicates the appropriate asset allocation at each time in the future until the date at which money in the fund is to be transferred to another vehicle. In effect, the fund has a base allocation for every period, which can be straightforwardly accommodated in the adaptive asset allocation formula. Let $X_{ib,t}$ represent the "base" allocation for time t as specified in the current policy. This allocation replaces the constant allocation given by $X_{if,0}$ in Equation 15:

$$X_{if,t} = \frac{X_{ib,t}\left(X_{im,t}/X_{im,0}\right)}{\sum_i X_{ib,t}\left(X_{im,t}/X_{im,0}\right)}. \tag{18}$$

Equation 15 is more general than Equation 18, which can be considered a special case in which $X_{ib,t} = X_{if,0}$ for every period t.

Figure 4 illustrates an adaptive target-date fund. It assumes that the fund was started in January 1976 with a glide path calling for 90 percent in stocks initially, with the allocation decreasing by a constant percentage each month until it reaches 10 percent in June 2009. Figure 4 shows the base allocations that would be implemented by a traditional target-date fund, as well as the market proportions.

To convert this traditional fund to an adaptive target-date fund, we assume that the original glide path was chosen as appropriate when the market proportions in stocks and bonds are 60 percent and 40 percent, respectively. Using these proportions for the $X_{im,0}$ values in Equation 18, we can see the allocations for the adaptive target-date fund. Because the glide path is assumed to be optimal when the market proportions are 60/40, the adaptive proportions and the policy glide path coincide when stocks are 60 percent of the total value of bonds and stocks. Whenever the market proportion of stocks is below 60 percent, the fund's allocation to stocks is below that specified by the glide

Figure 3. Asset Allocations: Market and an Aggressive Balanced Fund

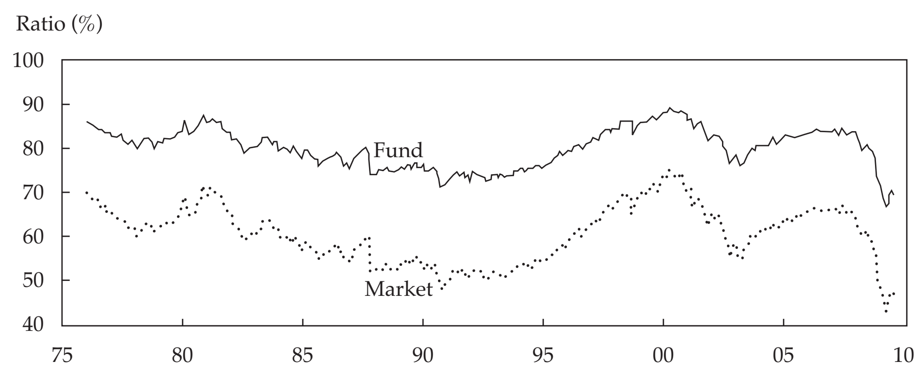
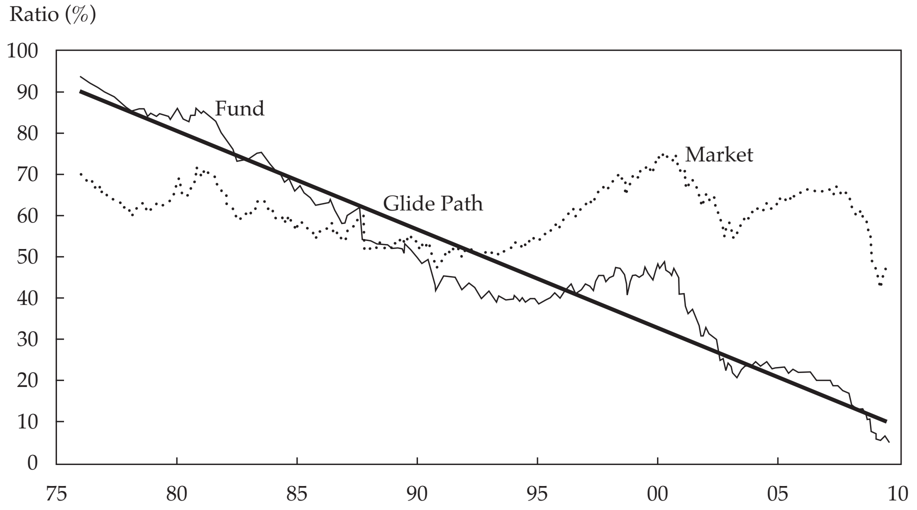


Figure 4. Asset Allocations: Market and an Adaptive Target-Date Fund



path. Conversely, when stocks represent more than 60 percent of the value of the market, the fund's allocation to stocks is greater than that called for by the glide path.

Conclusion

If my arguments have merit, a number of changes should be made by the investment industry.

First and foremost, more data will need to be made available about the market values of the securities in each of the benchmarks designed to represent major asset classes. Most such indices are computed by third parties. For example, Barclays Capital and Wilshire Associates calculate the two indices used in this article. Recent and historical monthly *returns* for most popular indices may be difficult but not totally impossible to obtain from such providers.[24] But obtaining data for the market *values* of the securities in an index is much harder. Clearly, the index provider has such information.[25] In cases where returns for the index are computed by using a subset of the securities in the represented universe, the provider should still have sufficient information to provide an estimate of the total market value of the class.

It is unclear whether the lack of widespread availability of asset class market values is a result of providers' desires to recover the costs of obtaining such information through subscription fees, a lack of sufficient interest on the part of investors and investment managers, or both. Of course, the thesis of this article is that asset market values are highly relevant for any decision concerning asset allocation, whether made episodically or adjusted routinely by using a procedure such as the one I have proposed. If more investment managers adopt my proposed procedure, market value data may become more widely available.

I cannot easily understand why funds do not routinely compare their asset allocations with current market proportions in order to ensure that any differences are commensurate with differences between their circumstances and those of "the average investor." Yet this comparison is rarely done. Perhaps the lack of easily obtained data on market values is the cause, with the absence of such comparisons the effect. Alternatively, the situation may be the reverse, with the lack of available data on market values caused by insufficient investor interest.

I would hope that readers of this article would request (or demand) that those who provide benchmark indices make the corresponding market values equally available. For publicly offered mutual funds or exchange-traded funds, this provision could be encouraged or required by regulatory authorities.

Second, it would be useful if institutional investors, armed with appropriate market value data, would at least compute the asset allocations that would result from an adaptive policy of the type described here. These computations could inform discussions between staff members and those charged with oversight, such as the members of investment committees and boards. In time, more investment organizations could become sufficiently comfortable with adaptive procedures to substitute them for the contrarian policies associated with the traditional approach.

Third, some mutual fund companies might offer adaptive balanced funds and/or adaptive target-date funds. In time, such vehicles might attract enough investors to represent a significant part of the market. A good first step would be to offer a balanced index fund designed to truly represent the mix of stocks and bonds in the United States. As shown in Figure 2, such a fund would differ significantly from such offerings as the Vanguard Balanced Index Fund. If successful, a representative balanced fund might pave the way for additional funds to follow adaptive asset allocation policies.

Although some will find the arguments in this article obvious and others will consider them radical, I hope that a sufficient number of readers will be convinced of the merit of these arguments to lead to changes in investment practice.

I thank Geert Bekaert of Columbia University, Steven Grenadier of Stanford University, Jesse Phillips of the University of California, Eamonn Dolan of C.M. Capital, Carlo Capaul of Julius Baer, and John Watson and Robert Young of Financial Engines, Inc., for their helpful comments.

This article qualifies for 1 CE credit.

Notes

1. Maginn, Tuttle, Pinto, and McLeavey (2007, p. 231).
2. CalPERS (2009).
3. CalPERS (2009).
4. U.S. Department of Labor (2009).
5. More specifically, the pertinent regulation provides for four types of QDIAs:

   > (1) A product with a mix of investments that takes into account the individual's age or retirement date (an example of such a product could be a life-cycle or targeted-retirement-date fund); (2) an investment service that allocates contributions among existing plan options to provide an asset mix that takes into account the individual's age or retirement date (an example of such a service could be a professionally managed account); (3) a product with a mix of investments that takes into account the characteristics of the group of employees as a whole, rather than each individual (an example of such a product could be a balanced fund); and (4) a capital preservation product for only the first 120 days of participation (an option for plan sponsors wishing to simplify administration if workers opt out of participation before incurring an additional tax).

   See U.S. Department of Labor (2009).
6. Appell (2009, pp. 11, 14).
7. Vanguard (2009).
8. Vanguard (2009).
9. Appell (2009, p. 12).
10. Fidelity Investments (2008, pp. 40–41).
11. Fidelity Investments (2008, p. 11).
12. Fidelity Investments (2009).
13. Investopedia.com, s.v. "Contrarian" (www.investopedia.com/terms/c/contrarian.asp).
14. This observation is true under the assumption made throughout the article that no asset proportions are negative. Funds with policies involving negative proportions (e.g., leverage) may buy relative winners and sell relative losers. For example, consider a fund with a policy of investing 150 percent in stocks and –50 percent in short-term bonds (loans). Assume that the initial position is $150 in stocks and –$50 in bonds, giving a net worth of $100. If the value of the stock portfolio doubles, the new position will equal $300 in stocks and –$50 in bonds, giving a net worth of $250. The new proportions will be 120 percent in stocks and –20 percent in bonds. To restore the fund to its policy proportions will require purchasing more stocks and selling more bonds (i.e., borrowing more money). Such a fund will thus purchase relative winners and sell relative losers.
15. That is, policies with non-negative asset proportions.
16. Maginn, Tuttle, Pinto, and McLeavey (2007, pp. 233–234).
17. Throughout the article, I use the "full-cap" version of the index rather than the "float-adjusted" alternative that uses weights for the securities based on estimates of the number of shares likely to be available for trading.
18. Each annualized monthly average is equal to 12 times the corresponding monthly average value.
19. For an early discussion of the need to account for changes in market values when making forecasts, see Rosenberg and Ohlson (1976).
20. An excellent example is described in EnnisKnupp (2009).
21. A mean–variance approach could be designed in which the asset allocation policy is characterized by a given ratio of the investor's risk tolerance to that of the market as a whole. I explored this possibility earlier (Sharpe 1990). To be practical, the reverse optimization procedure needs to be consistent with an equilibrium in which there are restrictions and/or costs associated with negative holdings—an issue that I have previously discussed (Sharpe 1991).
22. Full disclosure: I co-founded an investment management and advisory company (Financial Engines, Inc.) that uses asset market values and reverse optimization at least once a month to adjust forecasts of risks, returns, and correlations for such investment vehicles as mutual funds, followed by optimization analyses to make any appropriate changes in investor portfolios. Although no asset allocation policy per se is involved, the underlying philosophy is similar.
23. Dictionary.com, s.v. "Adapt" (http://dictionary.classic.reference.com/browse/adapt; retrieved 8 June 2009).
24. Some returns for the Barclays Capital U.S. Aggregate Bond Index can be obtained on the websites of mutual funds and exchange-traded funds that use it as a benchmark. Returns for the Wilshire 5000 Total Market Index can be obtained at www.wilshire.com/Indexes/calculator.
25. On its website, Wilshire (www.wilshire.com/Indexes/Broad/Wilshire5000/Characteristics.html) provides tables that include the total market values of the securities in its U.S. equity index at the end of the most recent month, but no historical data on market values are provided. The Barclays Capital website does not provide any data on the market values of the securities in the U.S. Aggregate Bond Index. I am grateful to both organizations for providing me with the historical data used in this article.
26. Fundamental Index is a registered trademark of Research Affiliates, LLC.

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